ABSTRACT. In the classical summability setting rates of summation have been introduced in several ways (see, e.g., [10], [21], [22]). The concept of statistical rates of convergence, for nonvanishing two null sequences, is studied in [13]. Unfortunately no single definition seems to have become the “standard” for the comparison of rates of summability transforms. The situation becomes even more uncharted when one considers rates of $A_i$ statistical convergence. For this reason various ways of defining rates of convergence in the $A_i$ statistical sense are introduced in [6].

In the present paper, using the concepts of [6], we study rates of $A_i$ statistical convergence of sequences of positive linear operators mapping the weighted space $C_{\frac{1}{1}}$ into the weighted space $B_{\frac{1}{2}}$ where $\frac{1}{1}$ and $\frac{1}{2}$ are weight functions satisfying the condition
\[
\lim_{|x| \to \infty} \frac{\frac{1}{2}(x)}{\frac{1}{2}(x)} = 0;
\]
and
\[
B_{\frac{1}{2}} := \{ f : f : \mathbb{R} \to \mathbb{R}, |f(x)| \leq M_f \frac{1}{2}(x) \text{ for all } x \in \mathbb{R} \},
\]
and
\[
C_{\frac{1}{k}} := \{ f : f \in B_{\frac{1}{k}}, f \text{ is continuous on } \mathbb{R} \},
\]
(here $M_f$ is a constant depending on $f$).

Note that the classical Korovkin type approximation theory may be found in [1], [4], [20] while its further extensions studied via $A_i$ statistical convergence may be viewed in [6], [7], [15].

Recall that the sequence $(x_n)$ is said to be $A_i$ statistically convergent to $L$ if, for every $\varepsilon > 0$;
\[
\lim_{n \to \infty} \frac{1}{|x_n|} \sum_{j} a_{nj} = 0
\]
where $A = (a_{ij})$ is a non-negative regular matrix (see, e.g., [2], [3], [9], [23]). The case in which $A = C_1$: the Cesáro matrix, $A_i$ statistical convergence reduces to statistical convergence [8], [11], [12].

1991 Mathematics Subject Classification. Primary 41A25, 41A36, 47B38; Secondary 40A05.

Key words and phrases. $A_i$ density, $A_i$ statistical convergence, sequence of positive linear operators, weight function, weighted space, modulus of continuity, the Korovkin theorem.

Section Number. 10 (Functional Analysis).
References


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