INVARIANT SUBSPACES IN SOME FUNCTION SPACES ON TANGENT SPACE

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Let G be a Lie group that acts transitively on a smooth noncompact manifold X. For any $g \in G$ and any function $f(x)$ on X, we put

$$(\pi(g)f)(x) := f(g^{-1}x).$$

A locally convex space $\mathcal{F}$ that consists of complex-valued functions on M will be called π-invariant if for any $f(x) \in \mathcal{F}$ and any $g \in G$ we have $\pi(g)f \in \mathcal{F}$ and $g \mapsto \pi(g)f$ is a continuous map from G to $\mathcal{F}$. Then the operators $\pi(g)\big|_{\mathcal{F}}$ define the quasi-regular representation of the group G on the topological vector space $\mathcal{F}$. A vector subspace $H \subseteq \mathcal{F}$ is called an *invariant subspace* if it is closed and π-invariant. One of the main problems of harmonic analysis on Lie groups is to describe all invariant subspaces for various Lie groups G, homogeneous manifolds X and various function spaces $\mathcal{F}$. The most thoroughly studied the case when X is a Riemannian symmetric space of non-compact type and G is the group of isometries of X (see [1], [2]).

Let M be a Riemannian symmetric space of non-compact type. Let $X = T_oM$ be a tangent space to the manifold M at some point $o \in M$. Denote by G the group of isometries of M and let K be the isotropy subgroup of the point o. The vector space $X = T_oM$ is an Abelian Lie group. Let $k_*\xi := (dk)_o\xi$ be the induced action of K on X where dk is the differential of k and $\xi \in X$. We can form semidirect product $G_0 = X \rtimes K$ with respect to this action of K on X. The group G_0 is called the Cartan motion group of X. The group G_0 acts transitively on X by $(\xi, k)x := \xi + k_*x$ where $(\xi, k) \in G_0$, $x \in X$.

In the paper we consider a description of invariant subspaces for the case where $X = T_oM$ is the tangent space to a Riemannian symmetric space M of rank 1 at the point o, the Cartan motion group G_0 acts on X, a function space $\mathcal{F} = C^k$, $k = 0, 1, \ldots, \infty$.

REFERENCES

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