On some operators with complex-valued continuous negative definite symbol

V.Knopova *

Keywords: Bessel potential spaces, pseudo-differential operator, sub-Markovian semigroup.

Mathematical Subject Classification: 26A33, 31C25, 35S99, 47D07.

Section Number: 12.

Consider the fractional power of an operator of the form

$$-A_{\pm} = -\psi(D_{x'}) \pm \frac{\partial}{\partial x_{n+1}}, \quad (x', x_{n+1}) \in \mathbb{R}^{n+1}_{0+},$$

where $\psi(D_{x'})$ is an operator with real continuous negative definite symbol ψ: $\mathbb{R}^n \to \mathbb{R}$.

Our aim is to prove that the operator $((-A_{\pm})^{\alpha}, D((-A_{\pm})^{\alpha})$ generates L_p-sub-Markovian semigroups, $1 < p < \infty$, $0 < \alpha < 1$, $D((-A_{\pm})^{\alpha})$ is the domain of $(-A_{\pm})^{\alpha}$ in $L_p(\mathbb{R}^{n+1}_{0+})$. It is possible to determined the domain $D((-A_{\pm})^{\alpha})$ in terms of the generalized Bessel potential spaces on the half-space $\mathbb{R}^{n+1}_{0+}$, if we pose some additional conditions on the function ψ.

To prove that the given operator generates an L_p-sub-Markovian semigroup, we apply the Hille-Yosida theorem, i.e. we proved that the domain of the operator is dense in $L_p(\mathbb{R}^n)$, and show that for some $\lambda > 0$ the boundary value problem

$$\lambda f + (-A_{\pm})^{\alpha} f = g, \quad f \in D((-A_{\pm})^{\alpha}), \quad g \in L_p(\mathbb{R}^{n+1}_{0+}),$$

is uniquely solvable for different boundary conditions (the choice of which depends on the domain). Note that the operator we consider is the Dirichlet operator since its symbol is continuous and negative definite.

The representations of the semigroups for Dirichlet and Neumann boundary conditions are also found.

[1] V.Knopova, "On some operators with complex-valued continuous negative definite symbol which are generators of L_p -sub-Markovian semigroups", in preparation;

[2] V.Knopova, "Semigroups generated by certain pseudo-differential operators in the half-space $\mathbb{R}^{n+1}_{0+}$", in preparation.

*V.M.Glushkov Institute of Cybernetics National Academy of Science of Ukraine; Address: 40, Acad.Glushkov Ave., 03187, Kiev, Ukraine, e-mail: *vic_knopova@yahoo.co.uk*