Let X be a locally compact Abelian separable group, $\Gamma(X)$ be the set of Gaussian distributions on X, $\text{Aut}(X)$ be the set of topological automorphisms of X. Consider the linear forms $L_1 = \alpha_1(\xi_1) + \cdots + \alpha_n(\xi_n)$ and $L_2 = \beta_1(\xi_1) + \cdots + \beta_n(\xi_n)$, $n \geq 2$, where $\alpha_j, \beta_j \in \text{Aut}(X)$, ξ_j are independent random variables with values in X and with distributions μ_j such that their characteristic functions do not vanish. By the classical Skitovich-Darmois theorem on the real line $X = \mathbb{R}$ the independence of L_1 and L_2 implies that all $\mu_j \in \Gamma(X)$ ([1]). The analogous result for the group $X = \mathbb{R}^m, m \geq 2$ was proved by Ghurye and Olkin ([1]). The corresponding result for the $\mathbf{a}$-adic solenoids $\Sigma_{\mathbf{a}}$ was proved by the author ([2]). The aim of the talk is to give the complete solution of the following problems ([3]).

Problem 1. To describe all locally compact Abelian groups X possessing the property: if ξ_j are independent random variables with values in X having the distributions μ_j with nonvanishing characteristic functions and α_j, β_j are arbitrary topological automorphisms of X, then the independence of L_1 and L_2 implies that all $\mu_j \in \Gamma(X)$.

Problem 2. To describe all locally compact Abelian groups X possessing the property: there exist $\alpha_j, \beta_j \in \text{Aut}(X)$ (not all $\alpha_j {\beta_j}^{-1}$ are equal) such that if ξ_j are independent random variables with values in X having the distributions μ_j with nonvanishing characteristic functions, then the independence of L_1 and L_2 implies that all $\mu_j \in \Gamma(X)$.

References

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