Positivity of real symmetric polynomials

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The following very general result could have *important implications* for Quantifier Elimination theory and algorithms (the symmetric case).

Corollary (see [3], p.177, Cor.2.1)

(a) A symmetric polynomial inequality (strict or not) of degree $d \geq 2$ holds on $\mathbf{R}^n$, if and only if, it holds for variables having at most $d/2$ distinct components (which may have multiplicities).

(b) A symmetric polynomial inequality (strict or not) of degree $d \geq 2$ holds on $\mathbf{R}^n_+$, if and only if, it holds for variables having at most[1] $[d/2]$ nonzero distinct components (which may have multiplicities).

This is a particular case of some more general facts from [3], and generalizes results from [1] (degree 3) and [2] (degree 4). First implications are explored in [4]–[6].

References

[1] M. D. Choi, T. Y. Lam and B. Reznick, Even symmetric sextics, *Math. Z.* **195** (1987), 559–580.

[2] W. R. Harris, Real even symmetric ternary forms, *J. Algebra* **222** (1999), 204–245.

[3] V. Timofte, On the positivity of symmetric polynomial functions. Part I: General results, *J. Math. Anal. Appl.* **284** (2003), 174–190.

[4] V. Timofte, On the positivity of symmetric polynomial functions. Part II: Lattice general results and positivity criteria for degrees 4 and 5 (preprint).

[5] V. Timofte, On the positivity of symmetric polynomial functions. Part III: Extremal polynomials of degree 4 (preprint).

[6] V. Timofte, Discriminants for positivity of real symmetric n-ary quartics (preprint).

[1] We write $[a]$ for the integer part of $a \in \mathbf{R}$.