Asymptotic one phase soliton type solutions to perturbed Korteweg-de Vries equation

Valeriy Hr. Samoylenko

Korteweg-de Vries equation is one of the most known nonlinear partial differential equation and it is a fundamental equation of modern physics. It was deduced in 1895 in order to mathematically describe so-called solitary waves on water discovered by Jone Scott Russel in 1834. The secondary discovery of this outstanding equation in 1965 is connected with attempt to solve famous Fermi-Pasta-Ulam problem. At present it is appeared as object for studying in different fields of physics.

During the last 35 years a lot of papers was devoted to consideration of different properties of solutions to the Korteweg-de Vries equation, specifically, finding its solutions in exact form mainly via inverse scattering transform approach. On the other hand, while considering processes with small variation of medium characteristics a problem of studying Korteweg-de Vries equation with varying coefficients and small parameters arises.

We consider Korteweg-de Vries equation with varying coefficients and small parameter at the higher derivative of the following form

$$u_{xxx} = a(x, \varepsilon) u_t + b(x, \varepsilon) uu_x.$$  

(1)

Functions $a(x, \varepsilon), b(x, \varepsilon)$ are assumed to be represented as

$$a(x, \varepsilon) = \frac{1}{\varepsilon^n} \sum_{k=0}^{\infty} a_k(x) \varepsilon^k, \quad b(x, \varepsilon) = \frac{1}{\varepsilon^n} \sum_{k=0}^{\infty} b_k(x) \varepsilon^k,$$

where $x \in \mathbb{R}^1; t \in (0; T); a_k(x), b_k(x) \in C^\infty(\mathbb{R}^1), k \geq 0; n$ is an integer.

Basing on small parameter approach we present algorithm of constructing asymptotical approximation for one phase soliton type solutions to problem (1). Specific peculiarity of the problem is a lot of additional problems connected with nonlinearity of unperturbed problem (when parameter $\varepsilon = 0$ in (1)), different type of asymptotics depending on integer $n$ and necessity of determining phase (curve of discontinuity) of a solution. We present also theorems on the order with that asymptotical solution satisfies the problem.