The Spectral Expansion on the Entire Real Line of Green's Function for a Three-Layer Medium in the Fundamental Functions of a Nonself-Adjoint Sturm-Liouville Operator

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Green's function $u(z, x)$ satisfies the Helmholtz equation with the delta function on the right-hand side and the radiation condition. The function u has a logarithmic singularity at the source position $M'(z', x')$. At the points of discontinuity of the coefficient in the Helmholtz equation, the function $u(z, x)$ satisfies the matching conditions for the function and its normal derivative. We consider the Helmholtz equation under the assumption that z, z', x, x' belong to the real axis R. We set that the coefficient in the equation depends on the one variable z and represents three complex constants.

To find the solution, we consider the Fourier expansion of u with respect to the variable x [of which the coefficient in the equation is independent]. Using the Cauchy theorem, we reduce the Fourier integral in the upper or the lower half of the complex plane of a spectral parameter to two integrals over the edges of the cuts passing through the ramification points.

We have thereby obtained the expasion of Green's function in the fundamental functions, which are finite on the entire real line solutions of the ordinary Sturm-Liouville equations with the complex coefficients.

The spectrum consists of two half-lines parallel to the real axis on the complex plane of a spectral parameter and going in the positive direction of the real axis.

The case of a three-layer medium which is considered in this paper is the generalization of the case of a two-layer medium which was considered in [1].

References

1. Saltykov, E.G., Differents. Uravn., 2002, vol. 38, no. 5, pp. 687-691.