A spectral theoretic approach to quantum integrability

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We prove that any n-dimensional Hamiltonian operator with pure point spectrum is completely integrable via self-adjoint first integrals [1]. This theorem is a consequence of the following result:

Proposition 1 *Let C be a sequence of real numbers. Then there exists an integrable n-dimensional Hamiltonian H with pure point spectrum, whose n commuting first integrals can be chosen to be self-adjoint, which realizes the sequence C as its point spectrum.*

We provide a constructive proof for this proposition. The definition of pure point spectrum we use is that of [2]. Another consequence of Proposition 1 is that given any closed set $\Sigma \subset \mathbb{R}$ there exists an integrable n-dimensional Hamiltonian which realizes it as its spectrum. Finally, we develop some non-trivial applications of our integrability criterion. For instance, a remarkable result is that:

Proposition 2 *For almost all Hamiltonians with pure point spectrum, its point spectrum is uniformly distributed.*

As discussed in [1], this proposition ensures that Berry's conjecture, as studied by physicists, holds for the class of Hamiltonians with pure point spectrum. Although extensive numerical research has been carried out, this is, to the best of our knowledge, the widest class of Hamiltonians for which this conjecture has managed to be rigourously proved.

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References

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