I.L.Bogolubsky

Laboratory of Information Technologies, Joint Institute for Nuclear Research, Dubna, near Moscow, Russia, Email: bogolubs@cv.jinr.ru

New localised solutions of physical importance in the S^2 extensions of the sine-Gordon equation

Abstract

We shall report on new topological localised solutions, their bound states and dynamical properties within classical easy-axis Heisenberg antiferromagnet model ("the $A3$ model") and its $U(1)$ gauge-invariant extension ("the $A3M$ model") in (1+1) and (2+1) Minkowski space-time respectively. These ($A3$ and $A3M$) are 3-component Lorentz-invariant sigma-models with values on the unit sphere S^2. The $A3$ model possesses $Z(2)$ and global $U(1)$ symmetries, the latter becomes local for the $A3M$ model.

The $A3$ model can be treated as complex-valued extension of the sine-Gordon equation, but contrary to well-known complexifications of the SG equations (e.g. Lund-Regge one), the $A3$ model is of physical importance and possesses kink (antikink) and breather localised solutions, which generalize the corresponding solutions of the sine-Gordon equation.

The main emphasis will be made on the 2-dimensional extended strings which have been found within the $A3M$ model. These stationary (time-independent) topological solitons have been found for values of topological indices ("charges") $Q_t = 1, 2$, using "hedgehog" ansatz for 3-component unit isovector field and "vortex" ansatz for the Maxwell field. Solitons exist for the values of dimensionless anisotropy parameter p such that $0 < p < p_{cr} \approx 0.4$, and are dynamically stable for all admissible p. The energy of solitons is less than $8\pi Q_t$, that is the energy of Belavin-Polyakov topological solutions in isotropic model; for $p \to p_{cr}$ soliton energy approaches $8\pi Q_t$ from below. For $p \to 0$ the soliton energy is about 1/3 of the Belavin-Polyakov value for given Q_t.

It is instructive to compare the $A3M$ topological solitons with Nielsen-Olesen vortices-strings found within the Abelian Higgs model. This comparison shows advantages of the $A3M$ solitons from the field theoretical viewpoint. The $A3M$ extended strings can be considered as an alternative to infinitesimally thin strings being considered in modern (super)string theory, in order to avoid possible ultraviolet divergencies of Feynman graphs caused by zero radius of strings.

34B15; 35Q51; 35Q60; 49S05; 51P05+81R40; 55Q40; 81T30

Key words: topological solitons, extended strings, vortices, sine-Gordon equation , hedgehog ansatz, Heisenberg antiferromagnet, easy-axis anisotropy.