Interaction Of Eigenvalues In Multi-Parameter Problems

Alexei A. Mailybaev and Alexander P. Seyranian

Moscow State University, Russia

E-mail: mailybaev@imec.msu.ru, seyran@imec.msu.ru

Section: 14. Mathematical Physics

1991 Mathematics Subject Classification: 15A18, 37G

Key words: interaction of eigenvalues, multiple eigenvalues, multi-parameter perturbation, bifurcation

We present a general theory of interaction of two eigenvalues of matrix operators on the complex plane depending on multiple parameters. Both complex and real eigenvalues are considered. Strong and weak interactions of eigenvalues, when one of the parameters is changed while increments of others remain constant, are distinguished based on a Jordan form of the matrix. Strong interaction of eigenvalues is characterized by a single eigenvector at the point of coincidence. It is shown that the strong interaction is described by hyperbolae with perpendicular asymptotes on the complex plane if the double eigenvalue λ_0 is complex, and by a parabola and straight line if λ_0 is real. Weak interaction of eigenvalues is characterized by two linearly independent eigenvectors at the point of coincidence. It is revealed that weak interaction is described by hyperbolae or a small elliptic bubble appearing from the point of the double eigenvalue λ_0 perpendicular to the plane of original interaction in case of real λ_0. If λ_0 is complex the interacting eigenvalues keep or interchange their main directions of motion on the complex plane before and after the weak interaction. It is emphasized that the presented theory of interactions gives not only qualitative, but also quantitative results on behavior of eigenvalues based only on the information at the initial point in the parameter space. Physical applications are considered.