Mathematical aspects of stratified fluid in a homogeneous gravity field

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We consider the system which describes small displacements of an exponentially stratified viscous fluid in the gravity field

$$\left\{ \begin{array}{c} \rho_* \frac{\partial u_1}{\partial t} - \mu \Delta u_1 + \frac{\partial p}{\partial x_1} = 0 \\ \rho_* \frac{\partial u_2}{\partial t} - \mu \Delta u_2 + \frac{\partial p}{\partial x_2} = 0 \\ \rho_* \frac{\partial u_3}{\partial t} - \mu \Delta u_3 + g\rho \frac{\partial p}{\partial x_3} = 0 \\ \frac{g}{N^2 \rho_*} \frac{\partial \rho}{\partial t} - u_3 = 0 \\ \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = 0 \end{array} \right. . \tag{1.1}$$

For 1.1 , we establish the properties of existence and uniqueness of the solutions, the dissipation of energy of internal waves, and investigate the localization and structure of the spectrum of normal vibrations. From different points of view, we consider a remarkable analogy between stratified flows described by 1.1 , and rotational flows governed by the system 1.2 :

$$\left\{ \begin{array}{c} \frac{\partial \overrightarrow{u}}{\partial t} + \overrightarrow{\omega} \times \overrightarrow{u} - \mu \triangle \overrightarrow{u} + \bigtriangledown p = 0 \\ \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = 0 \end{array} \right. . \tag{1.2}$$

References

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