

Codes with one check symbol, their error detection capabilities and n -ary quasigroups

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A check digit system \mathfrak{C} with one check character is a systematic error detecting code over an alphabet Q which arises by appending a *check digit* a_{n+1} to every word $a_1 a_2 \dots a_n \in Q^n$, i.e. $a_{n+1} = g(a_1, a_2, \dots, a_n)$ ([3]). We shall call the code \mathfrak{C} an n -ary code (Q, g) .

A nonempty set Q with n -ary operation f such that in the equation $f(x_1, x_2, \dots, x_n) = x_{n+1}$ knowledge of any n elements of $x_1, x_2, \dots, x_n, x_{n+1}$ uniquely specifies the remaining one is called an n -ary quasigroup [1].

If in an n -ary code (Q, g) the operation g is an n -ary quasigroup operation, then this code will be called an n -quasigroup code (Q, g) .

Any n -ary code (Q, g) detects all single errors if and only if it is an n -ary quasigroup code.

Various check character systems based on n -ary quasigroups are constructed, their error detection capabilities are researched ([2]).

The automorphism groups of n -ary quasigroups that correspond to the ISSN code, the EAN code and the UPC code are described.

Errors of some types, that the system of the serial numbers of German banknotes can not detect, are listed.

References:

[1] V.D. Belousov: *n -Ary quasigroups*, Ştiinţa, Kishinev, 1972 (in Russian).

[2] G.L. Mullen and V. Shcherbacov, *Properties of codes with one check symbol from a quasigroup point of view*, Izvestiya AN RM. Matematica. No 3, 2002, p. 71-86.

[3] R.-H. Schulz: *Check Character Systems and Anti-symmetric Mappings*. H. Alt(Ed): Computational Discrete Mathematics, LNCS 2122, p. 136-147, 2001.