ALGEBRAIC COMPACTNESS OF $\prod \mathbf{M}_\alpha / \bigoplus \mathbf{M}_\alpha$

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In this note, we are working within the category $R\mathbf{Mod}$ of (unitary, left) R-modules, where R is a **countable** ring. It is well known (see e.g. Kiełpiński & Simson (1975), Theorem 2.2) that the latter condition implies that the (left) pure global dimension of R is at most 1. Given an infinite index set A, and a family $M_\alpha \in R\mathbf{Mod}$, $\alpha \in A$ we are concerned with the conditions as to when the R-module

$$\prod / \coprod = \prod_{\alpha \in A} M_\alpha / \bigoplus_{\alpha \in A} M_\alpha$$

is or is not algebraically compact. There are a number of special results regarding this question and this note is meant to be an addition to and a generalization of the set of these results. Whether the module in the title is algebraically compact or not depends on the numbers of algebraically compact and non-compact modules among the components M_α. One of the results is as follows:

Proposition 5. *Let* $|A| > \max(2^{|R|}, 2^{\aleph_0})$ *and* $\forall \alpha \in A$, M_α *is not algebraically compact. Then* $\prod / \coprod$ *is not algebraically compact.*

[4] R. Kiełpiński & D. Simson, On pure homological dimension, *Bulletin de L'Acad. Polon. Sci, Sé. Math.*, **23**(1975), No.1, 1–6.