Generalized Fitting modules and rings

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An R-module M is called strongly Hop...an (resp.strongly co-Hop...an) if for every endomorphism f of M the chain $Kerf \mu Kerf^2 \mu$ ::::: (Resp. $Imf \P Im f^2 \P$ :::) stabilizes.An R-module M is Fitting (i.e. M satisfay Fitting's lemma for every endomrphism f of M) if and only if M is a strongly-Hop...an and strongly co-Hpp...an The classe of strongly Hop...an (resp. co-Hop...an) modules lies properly between the class of Noetherian (Resp. Artinian) and the class of Hop...an (resp. co-Hop...an) modules. For a semiprojective (Resp. seminijective) module, M, we have: M is strongly co-Hop...an (resp.strongly Hop...an) if and only if M is Fitting .Some structure theorems are obtained. The strongly Hop...an (Resp.strongly co-Hop...an) rings are also investigated. A new characterization of the rings R such that every ...nitely generated R-module are Fitting is obtained. A version of Hopkins-Levitzki (Resp. Kaplanski) theorem for Artinian rings are proved. Namely, a strongly co-Hop...an artinian ring (Resp. Banach algebra) is strongly Hop...an (Resp. algebraic). Also we prouve for a commutative ring R: R[X] is strongly Hop...an if and only if A is strongly Hop...an.

References

[AZU ¡ 54] G. Azumaya, Strongly ¼-regular rings J. Fac. Sci. Hokaido Univ. 13 (1954), 34-39.

[DIS ¡ 76] F. Dischinger, Sur les anneaux fortement ¼-réguliers, C.R. , Acad. Sci. Paris, Sér. A, 283 (1976) 571-573.

[DRA ¡ 01] M.P. Drazin, Generalizations of Fitting's Lemma in arbitrary associative rings, Communications in Algebra, 29(8), 3647-3675 (2001).

[GHA ¡ 02] A. Ghorbani and A. Haghany, Generalized Hop...ans modules, Journal of Algebra 255 (2002) 324-341.

[HVE ¡ 01] A. Haghany and M.R. Vedadi, Modules whose Injective Endomorphisms Are Essential, Journal of Algebra 243, 765-779 (2001).

[KAP ¡ 50] I. Kaplansky, Topological representations of algebras, II, Trans. Amer. Math. Soc. 68 (1950), 62-75.

[KAS ¡ 82] F- Kasch, Modules and rings, in: London Mathematical society Monograph No. 17, Academic Press London (1982).

[SIT ¡ 74] A.M. Sinclair and A.W. Tullo, Noetherian Banach algebras are …nite dimensional. Math. Annalen, 211, (1974), 151-153.

[XUJ ¡ 95] Jinzoug Xu, When is a self-injective ring zero-dimensional?, in: Zero-dimensional commutative rings Ed. by D.F. Anderson and D.E. Dobbs, Lecture Notes in Pure and Applied math. vol 171, Marcel Dekker, Inc. New York (1995).

[VAR ¡ 92] K.Varadarajan, Hop…an and co-Hop…an objects, Publ. Mat. 36, (1992), 293-317.

[VAR ¡ 00] K.Varadarajan, Some Recent Results on Hop…city, co-Hop…city and related properties, in: International Sympossium on Ring theory, Ed. by F.Berkenmeier, Jae Keol Park and Young Soo Park., Birkhäuser (2000).