

Recently, B.-Y. Chen studied warped products which are CR -submanifolds in Kaehler manifolds and established general sharp inequalities for CR -warped products in Kaehler manifolds [1], [2].

Afterwards, I. Hasegawa and the present author [3] obtained a sharp inequality for the squared norm of the second fundamental form (an extrinsic invariant) in terms of the warping function for contact CR -warped products isometrically immersed in Sasakian manifolds.

Recently, we improved the above inequality for contact CR -warped products in Sasakian space forms.

Theorem [4]. *Let $\widetilde{M}(c)$ be a Sasakian space form and $M = M_1 \times_f M_2$ an n -dimensional contact CR -warped product submanifold, such that M_1 is a $(2\alpha+1)$ -dimensional invariant submanifold tangent to ξ and M_2 a β -dimensional C -totally real submanifold of $\widetilde{M}(c)$. Then the squared norm of the second fundamental form of M satisfies*

$$\|h\|^2 \geq 2\beta[\|\nabla(\ln f)\|^2 - \Delta(\ln f) + 1] + \alpha\beta(c + 3),$$

where Δ denotes the Laplace operator on M_1 .

Some applications are derived. In the present paper, we classify contact CR -warped products in Sasakian space forms which satisfy the equality case identically.

References

- [1] B.Y. Chen, *Geometry of warped product CR -submanifolds in Kaehler manifolds*, Monatsh. Math. **133** (2001), 177-195.
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- [3] I. Hasegawa and I. Mihai, *Contact CR -warped product submanifolds in Sasakian manifolds*, Geom. Dedicata **102** (2003), 143-150.
- [4] I. Mihai, *Contact CR -warped product submanifolds in Sasakian space forms*, Geom. Dedicata, to appear.