A Geometrical Approach to Quantum Holonomic Computing Algorithms

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ABSTRACT. The work continues a presentation of modern quantum mathematics backgrounds started in [3]. A general approach to quantum holonomic computing based on geometric Lie - algebraic structures on Grassmann manifolds and related with them Lax type flows is proposed. Making use of the differential geometric techniques like momentum mapping reduction, central extension and connection theory on Stiefel bundles it is shown that the associated holonomy groups properly realizing quantum computations can be effectively found concerning diverse practical problems. Two examples demonstrating 2-form curvature calculations important for describing the corresponding holonomy Lie algebra are presented in detail.

Keywords: quantum computers, quantum algorithms, dynamical systems, Grassmann manifolds, symplectic structures, connections, holonomy groups, Lax type integrable flows

The unitary mapping in a Hilbert space $\mathcal{H}^{(n+k)}$

$$U_f : \sum_{x\in\mathbb{Z}_+} \alpha_x|(x),(a) > \mapsto \sum_{x\in\mathbb{Z}_+} \alpha_x|(x),(a)\oplus(f(x)> \tag{0.1}$$

for all $\alpha_x \in \mathbb{C},\ x \in \mathbb{Z}_+$, is called a quantum computation for a mapping $f : \mathbb{R} \to \mathbb{R}$. One sees that the unitary operator $U_f : \mathcal{H}^{(n+k)} \mapsto \mathcal{H}^{(n+k)}$ counts all values of pairs $|(x),(f(x)) >$, $x \in \mathbb{R}$, which are exactly searched for.

Algorithms like that above can effectively solve, as is well known [4], the following important for applications problems not solvable generally in a reasonable time by usual classical computers:

i) factorization of a large integer $x \in \mathbb{Z}_+$ by its primes (P. Shor, 1994) and application it to encrypting messages encoded via the RSA system;

ii) search or sorting algorithm for finding an item in structured and unstructured data sets (L.K. Grover, 1996; T. Hogg, 1997);

iii) fast discrete Fourier transform (P. Shor, 1994);

iv) finding minimal periods of periodic functions (P. Shor, 1994; A. Kitayev, 1995) and other, and further controlling its action on information data vector from the proper quantum computation medium. Some examples which were recently treated by means of the Quantum Computing Algorithms are discussed:

1. Public key Cryptography: (RSA)-cryptosystem;
2. Holonomic Quantum Computations: a) Two-mode quantum-optical model, b) Lax-type flows model

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