Best Mathematical Models for Dynamical Systems

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We propose a new method for the best representation of dynamical nonlinear systems.

The proposed approach develops some ideas from our published works [1, 2] and is based on the best constrained approximation of mapping $\mathcal{F}$ in probability spaces by polynomial operator $\mathcal{P}_r$ of degree r. The operator $\mathcal{P}_r$ is designed from matrices of a special form. As a result, the approximant preserves the causality principle [3] and minimizes the mean square difference between a desired output $\mathcal{F}(\mathbf{x})$ and the output $\mathcal{P}_r(\mathbf{y})$ of the approximating model $\mathcal{P}_r$.

It is supposed that the observable input $\mathbf{y}$ represents an idealized input $\mathbf{x}$ contaminated with noise. Unlike the known approaches to the modelling of nonlinear systems, it is not assumed here that $\mathbf{x}$ and $\mathbf{y}$ can be presented as analytical expressions. The inputs and outputs of the system under consideration are elements of the probability spaces and therefore relationships between them are assumed to be given by some covariance matrices only. Another difference is that we consider the *optimal* causal model of nonlinear systems. In other words, the model that we provides guarantees the smallest associated error in the entire class of models under consideration.

An optimal model of the nonlinear system is provided here for the first time in the wide class of polynomial operators $\mathcal{P}_r$ while preserving the principle of physical realisability, causality.

We present the model of the nonlinear system, reformulate and extend the heuristic definition of causality and show how the model is adjusted to the causality concept. In particular, we define so called (δ, ε)-causality which is closer to realistic conditions than the earlier notion of 'idealized' cuasality. To satisfy the causality concept, the model is reduced to a representation by matrices of special form.

We provide a constructive solution to the problem, i.e. we obtain the equations for the matrices which determine the optimal model $\mathcal{P}_r^0$. We also establish the error equation associated with $\mathcal{P}_r^0$. It is shown that the model has a degree of freedom, the degree r of the operator $\mathcal{P}_r$. In particular, we establish that the error is decreased if the degree r of $\mathcal{P}_r^0$ is increased.

Results of numerical simulations are also presented.

References

[1] P.G. Howlett, A. Torokhti, 'A methodology for the numerical representation of non-linear operators on noncompact sets', *Numer. Funct. Anal. and Optimiz.*, **18**, no. 3 & 4 (1997), pp. 343–365. MR **98i:**47065.

[2] P.G. Howlett, A.P. Torokhti and C.E.M. Pearce, 'The modeling and numerical simulation of causal non-linear systems', *Nonlinear Analysis: Theory, Methods & Applications,* **47 (8)**, pp. 5559-5572, 2001.

[3] I.W. Sandberg, 'Time-Delay Polynomial Networks and Quality of Approximation,' *IEEE Trans. CAS. Part 1, Fundamental Theory and Applications,* **47**, pp. 40 - 49, Jan. 2000.

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