

Verification of Hyperbolicity and Structural Stability

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The aim of the talk is substantiation of a constructive method for verification of hyperbolicity and structural stability of discrete dynamical systems. The main tool here is a symbolic image [1] which is a directed graph constructed by a finite covering of the projective bundle. Hyperbolicity is tested by calculation of the Morse spectrum (the limit set of Lyapunov exponents of pseudo trajectories) which can be found for a given accuracy by the symbolic image [2]. If the Morse spectrum does not contain 0, then the chain recurrent set is hyperbolic and the system is Ω -stable.

To verify the transversality condition, we consider the symbolic image of the complementary differential $\widehat{D}f(x) = ((Df(x))^*)^{-1}$ on the projective bundle. A diffeomorphism f is shown to be structurally stable if and only if the Morse spectrum does not contain 0 and for the complementary differential there is no connection $CR^+ \rightarrow CR^-$. Here CR^+ and CR^- are the positive and negative chain recurrent sets on the projective bundle so that the Morse spectrum of $\widehat{D}f$ on CR^+ and CR^- is positive and negative, respectively. These conditions are verified by an algorithm based on the symbolic image of the complementary differential. The proofs are available in [3].

References

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