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On the Quadratic Boundedness of Motions of Uncertain Discrete System

One of the most actively developed areas in recent years is the dynamics of uncertain discrete-time systems (see [1] and the references therein). The objective of the given study is to establish the sufficient boundedness conditions for the uncertain descrete-time dynamical system.

Consider a mechanical system with a finite number of degrees of freedom whose dynamics is described by the defference equations

$$x(\tau+1) = Ax(\tau) + Gw(\tau, x(\tau)), \tag{1}$$

$x(t_0) = x_0 \in R^n$, $\tau \in \mathcal{T}_\tau = \{t_0 + k, k = 0, 1, 2, \dots\}$, $t_0 \in R$. The variable $x \in R^n$ describes the state of the system (1). The $n \times n$ matrices A and G are constant and known, and $w\colon \mathcal{T}_\tau \times R^n \to R^n$ is a vector function describing the nonlinearities of the system (1) and / or the perturbations acting on the system. The information of this perturbations is incomplete. However, it is assumed that they are uniformely bounded, i.e., $\|w(\tau, x)\| \leqslant 1$ for all $(\tau, x) \in \mathcal{T}_\tau \times R^n$.

The definition of quadratic boundedness of the motions of the system (1) is similar to that for the continuous case.

Definition. The motions of system (1) are called quadratically bounded if there exists a symmetric positive definite matrix $P \in R^n$ such that for all $x_0 \in R^n$ and $w \in \mathcal{F}$ there exists the constant $\eta = \eta(x_0) > 0$ such that $x(\tau) \in \mathcal{E}_P^\eta$ for any $\tau \geq \tau_0$, $\tau \in \mathcal{T}_\tau$. Here $\mathcal{E}_P^\eta = \{x \in R^n\colon\ x^{\mathrm{T}} P x \leq \eta\}$ and $\mathcal{F}$ is the class of all admissible uncertainties in system (1).

The following statement is valid.

Theorem. *Let for system (1) there exist a positive constant δ and a symmetric positive definite matrix P such that the inequality*

$$A^{\mathrm{T}}PA - P + (\alpha + \|G^{\mathrm{T}}PG\|)P - \delta^{-1}A^{\mathrm{T}}PGG^{\mathrm{T}}PA \leqslant 0. \tag{7}$$

holds. Then the motions of system (1) is quadratically bounded.

Analogous boundedness conditions are also established via vector Lyapunov function. The example is given.

1 Martynyuk, A.A. Analysis of stability of discrete systems. *Int. Appl. Mech.* 7(36) (2000) 3–35.