Dynamics of a plane endomorphism for Hopf bifurcation: a computer-assisted study

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Consider a two-parameter family of map of the extended plane $\tilde{R}^2 = R^2 \cup \infty$ into itself

$$T_{\lambda,\mu} : \qquad \begin{array}{rl} x_1 = & (1-\lambda)\, x + \lambda\mu y(1-y), \\ y_1 = & (1-\lambda)\, y + \lambda\mu x(1-x), \end{array} \tag{1}$$

where λ, μ — real parameteres and the infinity is supposed to be a fixed point for $T_{\lambda,\mu}$.

It is known that a bifurcation of a fixed point with complex eigenvalues leads to the occurence of an invariant circle which is smooth for parameters values in a neighborhood of the bifurcation point. The fixed point changes its stability type when the absolute value of the eigenvalue crosses a unit circle, which can be written as $e^{i2\pi p/q}$, with (p,q) in lowest terms. It is said that p/q-resonance occurs on the invariant circle if there is a pair of periodic orbits, one consisting of saddles and the other of sinks, being the rotation number of these periodic orbits is p/q.

By V.Arnold's method a map on R^2 can be written as a function of one complex variable and two real parameters can be written as a single complex parameter. So, the map has a form $f_\nu(z) = \nu z + O(|z^2|)$, where ν and $\bar{\nu}$ are eigenvalues at the fixed point $z = 0$, being the point on the unit circle denoted as ν_0.

The case $q \geq 5$ is called weak resonance. A pair of periodic orbits exists on the invariant circle for parameters values lying in the narrow cusped region (resonance horn), emanating from ν_0. [2] Arnold's method is applicable for a parameter when the invariant circle is smooth. In other case we need a computer simulation.

The strong resonance, $q = 1, 2, 3, 4$ exhibit rather different behaviour, the details of which are not yet fully understood. In this situation Arnold's method is not applicable as well.

The proposed approach is based on results obtained in [1] and combines both numerical and analytical methods of research. The map (1) has been investigated both in the case of weak and strong resonance. It has been shown that a transition to chaotic regime is a result of recurring Hopf bifurcation.

References

[1] N.Ampilova. Numerical exploration of a behaviour of invariant curves in a vicinity of fixed point for Gardini map.(in Russian)/ Nonlinear dynamical systems./Ed. G.A.Leonov.v.1. St.Petersburg, 1997. p.5-13.

[2] D.G.Aronson, M.A.Chory, G.R.Hall and R.P.McGehee. Bifurcation from an invariant circle for two-parameter families of maps of the plane: a computer-assisted study. Commun.Math.Phys. 83, 3(1982), p.303-354.