The following problem for a dissipative non-autonomous hyperbolic equation in a bounded domain Ω is considered:

$$\frac{\partial^2 u}{\partial t^2} + \gamma \frac{\partial u}{\partial t} = \Delta u - f(u,t) + g(x,t), \qquad u\Big|_{\partial\Omega} = 0, \tag{1}$$

$$u\Big|_{t=\tau} = u_\tau(x) \in H_0^1(\Omega), \tag{2}$$

$$u_t\Big|_{t=\tau} = p\Big|_{t=\tau} = p_\tau(x) \in L_2(\Omega). \tag{3}$$

Here $\gamma > 0$ is a dissipation coefficient, the nonlinearity $f(u,t)$ is C^1-smooth and $|\partial f/\partial u| \leqslant L$, $g(x,t) \in L_2^{\mathrm{loc}}(\mathbb{R}, L_2(\Omega))$ is an external force. In the special case $f(u,t) \equiv L \sin u$, equation (1) is the sine–Gordon equation.

Let λ_k, $0 < \lambda_1 < \lambda_2 \leqslant \ldots \to +\infty$, be eigenvalues of the operator $-\Delta$ in the domain Ω with the Dirichlet boundary conditions.

Theorem 1 *Let $\lambda_{N+1} - \lambda_N > 4L$ hold for some N and let γ be sufficiently large. Then, in an extended phase space, there exists an invariant finite-dimensional Lipschitz integral manifold M that exponentially attracts as $t \to +\infty$ all the solutions of problem* (1)–(3).

The proof of the theorem is based on a construction of a new scalar product (equivalent to the initial one) in the phase space $H = H_0^1(\Omega) \times L_2(\Omega)$, with respect to which the so-called *gap property* is satisfied. Then the construction of integral manifolds with exponential tracking is carried out similar to [1].

Additionally the following questions for problem (1)–(3) are studied:

- behavior of solutions when $t \to -\infty$;
- explicit construction (see [2]) of *semiinvariant* integral manifolds with exponential tracking;
- dependence of integral manifolds on the parameter in a case when f and g regularly depend on this parameter;
- averaging of integral manifolds in a case when the right-hand side of the equation has rapidly oscillating in t terms.

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References

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[2] *Chepyzhov, V. V., Goritsky, A. Yu.*, Explicit construction of integral manifolds with exponential tracking, *Applicable Analysis,* 1999, vol. 71, nos. 1–4, pp. 237–252.