V. G. Romanov
Sobolev Institute of Mathematics,
Acad. Koptyug prosp., 4, 630090, Novosibirsk, Russia,
e-mail: romanov@math.nsc.ru

A stability estimate of a solution in two-dimensional inverse problem of electrodynamics

The problem of determining three coefficients $c(x)$, $\sigma(x)$, $q(x)$ for a hyperbolic equation is considered. The inverse two-dimensional problem of electrodynamics is reduced to this problem.

Let $u = u(x,t)$, $x \in \mathbf{R}^2$, be the solution to the problem

$$u_{tt} + \sigma u_t - c^2(\Delta u + qu) = \delta(t)\,\delta(x\cdot\nu), \quad u|_{t<0} = 0, \tag{1}$$

where $\nu = (\nu_1, \nu_2)$ is a unite vector and $x \cdot \nu$ is the scalar product of $x = (x_1, x_2)$ and ν. Because the solution to problem (1) depends on ν we consider ν as a parameter of the problem, i.e., $u = u(x,t,\nu)$. Let a support of coefficients $c(x)-1$, $\sigma(x)$, $q(x)$ belongs to the disk $D := \{x \in \mathbf{R}^2 |\, |x - x^0| < r\}$ that is contained inside the half-plane $x\cdot\nu > 0$. Introduce $\tau(x,\nu)$ as the solution to the Cauchy problem for the eikonal equation

$$|\nabla\tau|^2 = c^{-2}(x), \quad \tau|_{x\cdot\nu=0} = 0 \tag{2}$$

and let $G(\nu)$ be the cylindrical domain $G(\nu) := \{(x,t)|\, x \in D, \tau(x,\nu) < t < T + \tau(x,\nu)\}$ where T is a positive number. Denote by $S(\nu)$ a lateral part of the boundary of $G(\nu)$, i.e., $S(\nu) := \{(x,t)|\, x \in \partial D, \tau(x,\nu) \le t \le T + \tau(x,\nu)\}$, $\partial D := \{x \in \mathbf{R}^2 |\, |x - x^0| = r\}$.

The following problem of determining coefficients $c(x)$, $\sigma(x)$, $q(x)$ is considered. Let function $\tau(x,\nu)$ be given on ∂D and traces of the solution to problem (1) and its normal derivative be given on $S(\nu^{(k)}) := S_k$ for tree different parameters $\nu = \nu^{(k)}$, $k = 1,2,3$, i.e.,

$$u(x,t,\nu^{(k)}) = f^{(k)}(x,t), \quad \frac{\partial}{\partial n}u(x,t,\nu^{(k)}) = g^{(k)}(x,t), \quad (x,t) \in S_k,$$
$$\tau(x,\nu^{(k)}) = \tau^{(k)}(x), \quad x \in \partial D, \quad k = 1,2,3. \tag{3}$$

The inverse problem is: find $c(x)$, $\sigma(x)$ and $q(x)$.

Under suitable conditions on a smoothness and a boundeness of $c(x)$, $\sigma(x)$ and $q(x)$ as well as a natural relation between T and the diameter of D conditional stability and uniqueness theorems for the inverse problem are stated.