The Influence of Viscosity on Oscillatory Phenomena for Stokes and Navier–Stokes equations

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We consider nonstationary (linearized) Stokes equations of hydrodynamics with periodic rapidly oscillating (with respect to the spatial variables) data and a small viscosity. The period of data oscillations is specified by a small positive parameter ε . We assume that, as $\varepsilon \to 0$, the coefficient of viscosity ν in these equations satisfies the relation $\nu \to 0$ and one of the following three conditions:

$$\frac{\nu}{\varepsilon^2} \to \infty, \tag{1}$$

$$\frac{\nu}{\varepsilon^2} \to \vartheta, \tag{2}$$

$$\frac{\nu}{\varepsilon^2} \to 0, \tag{3}$$

where ϑ is a given positive constant. Thus, the positive value ν depends on ε and is sufficiently small at small ε. In addition, the mutual smallness of ν and ε is controlled by one of relations (1)–(3) valid for $\varepsilon \to 0$.

We give averaged (limit) equations whose solutions determine approximations (leading terms) of the asymptotics of the solutions to the equations under consideration and estimate the accuracy of the approximations. These approximations and estimates shed light on the following interesting property of the solutions to the equations under consideration. When the viscosity is not too small [i.e., when condition (1) holds], the approximations contain no rapidly oscillating terms, and the equations under consideration asymptotically smooth the rapid oscillations of the data; thus, the equations are asymptotically parabolic. If the viscosity is small [condition (3) holds], the approximations for the velocity vector may contain rapidly oscillating terms with zero means, and the equations are asymptotically hyperbolic. Moreover, these terms are nonzero if the rapid oscillations of data are not potential in mean and can grow linearly with time. Thus, the considered equations with a very small viscosity have the property of space-time resonance with respect to spatial oscillations of the initial data. The methods used also apply to the nonstationary Navier–Stokes equations if the rapid oscillations of data are zero in mean and the viscosity is not too small.