ISOPERIMETRIC INEQUALITIES, PROBABILITY MEASURES AND CONVEX GEOMETRY

FRANCK BARTHE

The isoperimetric problem consists in finding subsets of a Riemannian manifold with prescribed volume and minimal boundary measure. The question may be asked for more general measures. This larger framework encompasses the above geometric problem as well as the probabilistic concentration of measure phenomenon. Moreover it allows transfers of ideas and techniques.

Product measures are of particular interest in probability theory (they correspond to independent variables). Given the isoperimetric profile of a measure μ, it is natural to ask what kind of isoperimetric inequalities hold for the product measures μ^n? We shall discuss recent progress in this direction (a few exact results, comparison theorems, infinite dimensional isoperimetric inequalities in relation with hypercontractive semigroups...).

Another intringuing class of measures is of interest in the geometry of convex sets and Banach spaces, namely log-concave isotropic probability measures on $\mathbb{R}^n$. Much less is known about their isoperimetric behaviour and very nice questions remain open.