SOME PROBLEMS RELATED WITH HOLOMORPHIC FUNCTIONS ON TUBE DOMAINS OVER LIGHT CONES

ALINE BONAMI

We will illustrate some of the activities of our network, and especially the fact that we emphasize within HARP the interplay between Euclidean Harmonic Analysis and its counterpart on Lie groups. The talk will be based on joint work with D. Buraczewksi (HARP, Wroclaw), E. Damek (HARP, Wroclaw), A. Hulanicki (HARP, Wroclaw) & Ph. Jaming (HARP, Orléans) on one side, and D. Békollé, G. Garrigós (HARP, Madrid) & F. Ricci (HARP, Pisa) on another side. It will use previous work of T. Tao & A. Vargas (HARP, Madrid).

Let us define the complex tube domain $\Omega = \mathbb{R}^n + i\Gamma \subset \mathbb{C}^n, n \geq 3$, where Γ is the forward light cone given by

$$\Gamma = \{y = (y_1, ..., y_{n-1}, y_n) \in \mathbb{R}^n : y_1 > \sqrt{y_2^2 + \cdots + y_n^2}\} . \tag{1}$$

The first question that we consider is related with real bounded functions which may be written as Poisson-Szegö integrals of boundary distributions. These last ones are given on the distinguished boundary of Ω, that is $\mathbb{R}^n$. Poisson-Szegö integrals are known to coincide with solutions of a second order system, called the Hua system. We show that all such functions are real parts of holomorphic functions as soon as they are smooth up to the boundary. This may be seen as the analogue of a well-known phenomenon for harmonic functions related to the invariant Laplacian in the unit ball in $\mathbb{C}^n$. Its proof involves harmonic analysis on the Heisenberg group.

The second question that we consider is the boundedness in different L^p spaces of the Bergman projection P, that is, the orthogonal projection onto the subspace of holomorphic functions in $L^2(\Omega)$. For $n = 3$, the projection P is unbounded for $p > 7$, and bounded for $2 \leq p < 5 + \varepsilon$, for some ε which is certainly not optimal. In fact, we show that the boundedness of P is equivalent with some problem of harmonic analysis in $\mathbb{R}^n$, related to functions with spectrum in Γ. This last problem is close from the cone multiplier problem in harmonic analysis, which has been the object of extensive work, and is still partly open. Some results of Laba-Wolff and Tao-Vargas in this direction allow us to obtain our best estimates at the moment.