A PROBABILISTIC APPROACH TOWARD SOME PROBLEMS IN VON NEUMANN ALGEBRAS

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Under the impulse of D. Voiculescu, probability theory was adapted to the context of operator algebras to give rise to free probability. Free probability is a probability theory for non-commutative variables with a notion of freeness, analogous to independence in classical probability. This notion is as well related with the usual notion of freeness in groups. Hence, fundamental problems, such as the problem of the isomorphism between free group factors, can be formulated in the context of free probability. On the other hand, free probability can hopefully borrow ideas from classical probability to solve them.

In the 1990's, D. Voiculescu proved that independent Gaussian random matrices become free as their size go to infinity. Hence, large random matrices became a source of intuition to construct interesting examples or counterexamples of algebras.

In this line of attack, D. Voiculescu proposed an entropy approach to study von Neumann algebras. The entropy theory which was subsequently developed takes its root in the key role played by large random matrices in free probability as well as classical Boltzmann/Shannon entropy. One of the goals of this theory would be to disprove the free group factors isomorphism problem by constructing an invariant of von Neumann algebras which would take different values for free group factors with different numbers of generators. A candidate, which generalizes the classical Minkowski dimension, was introduced by D. Voiculescu.

In this talk, we shall describe more precisely the above mentioned problems. Then, we will discuss how tools from classical probability such as large deviations or transport techniques can be used to complete the theory of free entropy.