By a famous theorem of E. Brieskorn from 1970, the intersection of the unipotent variety of an algebraic group of type A, D or E with a transversal slice to its singular locus is a rational double point of the same type as the group. This result was motivated by the similarities between the deformation theory of rational double points and the theory of invariant functions on the corresponding Lie algebra. For the same reason, it was conjectured that the simple elliptic singularities would appear in certain completions of affine Kac-Moody groups of type E. In a joint work with Peter Slodowy, we found a fine enough classification of conjugacy classes in holomorphic loop groups, which allowed us to actually prove this conjecture and even extend the results to affine Kac-Moody groups of other type than E. In those cases, some interesting non-isolated singularities with symmetry appear.