SUMSETS

IMRE Z. RUZSA

What can one say about a set of n numbers if we know that there are at most αn different values among the sums of pairs? The basic qualitative answer to this question is due to G. Freiman from the sixties: such a set is covered by a not too large generalized arithmetic progression. We survey subsequent efforts to quantitatively improve and generalize this result.

Generalizations go into two directions. First, we can consider sets in general groups rather than numbers. Abelian groups are quite well understood, but there are only a few isolated results for certain noncommutative groups. Next, instead of all sums we can consider only sums formed along the edges of a graph. If the graph is dense, we have structural results similar to the classical case. If the graph is sparse, new phenomena arise. This is connected to the Kakeya conjecture and some strange inequalities for entropies of sums of dependent random variables.