In this talk I will give an overview of some of the mathematics of large quantum systems, both the questions addressed and the techniques used. The focus will be on the stability and structure of large quantum systems. As a guiding example I will discuss a recent resolution of a long standing conjecture by F. Dyson, on the energy of a gas of charged particles asymptotically as the number of particles tends to infinity. This problem is closely related to one of the most celebrated results in mathematical physics, the Theorem on Stability of Matter. The analysis of large quantum systems is typically done by a controlled approximation by a mean-field effective model. For the charged gas, the effective model is a non-linear Schrödinger equation. The reduction to the effective model, in this case, requires a rigorous justification of Bogolubov’s famous pair theory for superfluidity.