THE GROTHENDIECK-TEICHMUELLER GROUP AND THE GALOIS THEORY OF THE RATIONAL NUMBERS

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In his ”Esquisse d’un Programme”, Grothendieck suggests -among other things- that one should try to give a description of the absolute Galois group of the rational numbers $\mathbb{Q}$, by studying its action on the geometric fundamental group of $\mathbb{Q}$-varieties. As ”good” candidates he proposes (categories of) moduli spaces of curves with marked points. And here, a special attention should be payed to the moduli of the Riemann sphere with n marked points. The Grothendieck-Teichmueller group GT is the simplest incarnation of this idea. A first observation here is that using ”tangetial base points” one can embed the absolute Galois group of $\mathbb{Q}$ in GT. But unfortunately (maybe interestingly?), we do not know how the two groups precisely compare, in particular whether the two groups are equal. By enlarging/modifying the category of $\mathbb{Q}$-varieties in discussion, one gets variants of GT (which might be equal to GT), reflecting more and more arithmetic of the rational numbers. My plan is to report about (some of) the research on this very exciting subject done over the past few years, but also about perspectives for the future.