THE SEMIADDITIVITY OF ANALYTIC CAPACITY AND THE PAINLEVÉ PROBLEM

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The Painlevé problem consists in finding a characterization in metric and geometric terms of removable singularities for bounded analytic functions in the complex plane. The notion of analytic capacity, introduced by Ahlfors in 1947, plays a crucial role in the study of this problem. Indeed, Ahlfors proved that a compact set is removable if and only if its analytic capacity is zero. In the 1950's, Vitushkin showed that analytic capacity is also useful in connection with problems of uniform rational approximation, and he conjectured that analytic capacity is semiadditive.

In this lecture we will survey recent results on analytic capacity and the Painlevé problem. In particular, we will describe the real variable characterization of analytic capacity in terms of the so called curvature of measures, which implies that analytic capacity is semiadditive.