1) $$(1-x)(1+x+x^2+\dots+x^n) = 1+x+x^2+\dots+x^n - (x+x^2+\dots+x^n+x^{n+1}) = 1-x^{n+1}$$

(eller via induktion)

2) $$\int_1^2 x^3 e^{x^2}\,dx \underset{\substack{u=x^2\\ du=2x\,dx}}{=} \frac{1}{2}\int_1^4 \underbrace{u}_{g}\,\underbrace{e^u}_{h'}\,du$$

$$= \frac{1}{2}ue^u\Big|_1^4 - \frac{1}{2}\int_1^4 \underbrace{1}_{g'}\cdot\underbrace{e^u}_{h} = 2e^4 - \frac{1}{2}e - \frac{1}{2}e^4 + \frac{1}{2}e = \frac{3}{2}e^4$$

3) $$f'(x) = \frac{1\cdot(1+x^2) - x\cdot 2x}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2} \overset{!}{=} 0 \Rightarrow x_\pm = \pm 1$$

är möjliga extrempunkter

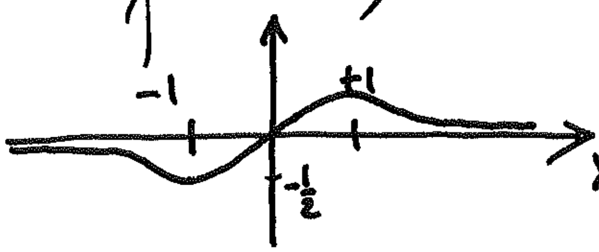
$f'(x) > 0 \Leftrightarrow x \in (-1,+1)$ och $f(x \to \pm\infty) \to 0$

$\downarrow$

Minimum i $x_- = -1$

Maximum i $x_+ = +1$

$$f(x_\pm) = \pm\frac{1}{2}$$



4) $$\frac{1}{x^2-3x+2} = \frac{1}{(x-2)(x-1)} = \frac{A}{x-2} + \frac{B}{x-1} = \dots \underset{(A+B=0,\ -A-2B=1)}{=}$$

$$\Rightarrow \int_3^4 \frac{dx}{x^2-3x+2} = \int_3^4 \frac{dx}{x-2} - \int_3^4 \frac{dx}{x-1} = \ln|x-2|\Big|_3^4 - \ln|x-1|\Big|_3^4$$

$$= \ln\frac{4}{3}$$

5) $y = e^{\lambda x} \to \lambda^2 - 2\lambda + 5 \overset{!}{=} 0 \to \lambda_{\pm} = 1 \pm \sqrt{-4} = 1 \pm 2i$

$\Rightarrow y_H(x) = e^x \left(A \cos 2x + B \sin 2x \right)$

$y_p(x) = C \sin x + D \cos x \Rightarrow$

$y_p'' - 2y_p' + 5y_p = 4y_p - 2y_p' = (4C + 2D) \sin x$
$+ (4D - 2C) \cos x$
$\overset{!}{=} \sin x + \cos x$

$\Rightarrow \begin{array}{l} 4C + 2D = 1 \\ 4D - 2C = 1 \end{array} \Rightarrow D = \frac{3}{10},\ C = \frac{1}{10}$

$\Rightarrow y(x) = e^x \left(A \cos 2x + B \sin 2x \right) + \frac{1}{10} \left(\sin x + 3 \cos x \right)$

6) $J = \int_0^\infty \left(\frac{x^2}{x^2+9} - \frac{x^2}{x^2+4} \right) dx = \int_0^\infty \left(1 - \frac{9}{x^2+9} - 1 + \frac{4}{x^2+4} \right) dx$

$= \lim_{\Lambda \to \infty} \left(4 \int_0^\Lambda \frac{dx}{x^2+4} - 9 \int_0^\Lambda \frac{dx}{x^2+9} \right) = \lim_{\Lambda \to \infty} \left(J_1(\Lambda) - J_2(\Lambda) \right)$

$J_1(\Lambda) \underset{x=2u}{=} 2 \int_0^{\frac{\Lambda}{2}} \frac{du}{u^2+1} = 2 \arctan \frac{\Lambda}{2}$

$J_2(\Lambda) \underset{x=3v}{=} 3 \int_0^{\frac{\Lambda}{3}} \frac{dv}{v^2+1} = 3 \arctan \frac{\Lambda}{3}$

$\Rightarrow J = 2 \cdot \frac{\pi}{2} - 3 \cdot \frac{\pi}{2} = -\frac{\pi}{2}$

7) $f(x) = \ln(1+x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \ldots$

$= 0 + x - \frac{x^2}{2} + \frac{x^3}{3} \mp \ldots$

$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots = \ln(1+1) = \ln 2$

8) $y(x) = e^{\lambda x} \rightarrow \lambda^2 - 5\lambda + 4 = 0 \rightarrow \lambda_{\pm} = \frac{5}{2} \pm \sqrt{\frac{25}{4} - 4}$

$\Rightarrow y_H(x) = A e^{4x} + B e^{x}$

$y_1'' - 5y_1' + 4y_1 \overset{!}{=} 2e^x \rightarrow y_1(x) = C \cdot x \cdot e^x$ ('Resonanz')

$\Downarrow \qquad (y_1' = Ce^x(x+1),\ y_1'' = Ce^x(x+2))$

$C\left(x+2-5(x+1)+4x\right) \overset{!}{=} 2 \Rightarrow C = -\frac{2}{3},\ y_1 = -\frac{2}{3} x e^x$

$y_2'' - 5y_2' + 4y_2 \overset{!}{=} 4x \rightarrow y_2(x) = Dx + E$

$\Downarrow \qquad (y_2' = D,\ y_2'' = 0)$

$-5D + 4Dx + 4E \overset{!}{=} 4x \Rightarrow D = 1,\ E = \frac{5}{4}$

$\Rightarrow y(x) = A e^{4x} + B e^x + \left(x + \frac{5}{4}\right) - \frac{2}{3} x e^x$

9) $\tan x = \frac{\sin x}{\cos x} = \frac{(x - x^3/3! + \dots)}{1 - x^2/2 + \dots} = (x - x^3/6 + \dots)(1 + \frac{x^2}{2} + \dots)$

$= x + \frac{1}{3}x^3 + \dots$

$\Rightarrow \lim_{x\to 0} \frac{(x - \tan x)}{x^3} = \lim_{x\to 0} \frac{(x - x - \frac{1}{3}x^3 + \dots)}{x^3} = -\frac{1}{3} \overset{!}{=} a = f(0)$

10) $\dot{x} = -3 \sin t \cdot \cos^2 t, \quad \dot{y} = 3 \cos t \sin^2 t$

$\Rightarrow L = \int_0^{2\pi} \sqrt{\dot{x}^2 + \dot{y}^2}\, dt = 3 \cdot \int_0^{2\pi} |\sin t \cdot \cos t|\, dt$

$= 3 \cdot 4 \cdot \int_0^{\pi/2} \sin t \cos t = 3 \cdot 2 \int_0^{\pi/2} \sin 2t\, dt$

$\underset{u=2t}{=} 3 \cdot \int_0^{\pi} \sin u\, du = -3 \cos u \Big|_0^{\pi} = 6$