Homework nr 5

Handed out October 30, 2007
To be handed in November 20, 2007

Part A Construction of Daubechies Orthogonal wavelet filter of length 6.

1. Write Z transform of lowpass filter h as $H(z) = (1 + \frac{1}{z})^3 P(\frac{1}{z})$ where P is a polynom of degree 2. It remains to find P.
2. Let $Q(z) = P(z)P(\frac{1}{z})$.
3. let q and b be the filters with Z transforms $Q(z)$ resp. $(1+z)^3(1+\frac{1}{z})^3$
4. Now find q so that $\{T^{2k}q\}_k$ and $\{T^{2k}b\}_k$ will be bi-orthogonal filter by solving a few linear equations. Use this to find Q
5. Find the polynomial P from knowing Q by solven equation on (2) above. (This is numerically the hardest part)
6. Finally find from this the lowpass filter h and the highpassfilter g

Part B. The continuous wavelet transform

1. Create a mat-lab function cwavtrans:

   [wcoeff] = cwavtrans(signal,scales)

   which computes the continuous wavelet transform of the real column vector signal using the complex morlet wavelet. over the scales given in the column vector scales.

2. apply this transform on some given signals guitar.wav and noisyguitar.wav
   given in the web directory

   http://www.math.kth.se/ jostromb/SF2702/

   (click for save, on the file, shown in this directory)

The complex morlet wavelet is given by the formula[

$$\psi(x) = \frac{d}{dx}\exp\{-x^2\}\exp\{iAx\}$$

where A is a positive number. We will choose $A = 5$.
The Gaussian function $\exp\{-x^2\}$ is rapidly decreasing as $|x|$ increases and we may with good accuracy approximate it by 0 when say $|x| \geq 6$.
Question: how large is its value for $x = 6$.
Let s_{max} be the maximal scale in the given input. Represent the functions

$$\frac{1}{\sqrt{s_j}}\psi(x/s_j)$$

sampled on integers on the interval $[-6s_{max}, 6s_{max}]$ as a matrix Ψ one column for each scale. Notify which row in the matrix that correspond to the point zero on the interval.
The derivative in the formula for the Morlet wavelet should be done by taking a simple difference between neighbouring sample points.
It now only remains to convolve the signal (a column vector) by each column vector in in the matrix Ψ. Keep track of where the "zero row" is in the Ψ .
Matlab has some function for convolution but the are rather slow. I plan to provide a faster function conv_fft(signa,matrix) on the course web site. It is a so called mex file conf_fft_float.xxxx (where xxx is specific for the operative system)
for Windows xxx=dll, for Sun:xxx=mexsol. for Linux xxx=mexsglx.
Those file hidden in the SF2702 directoroy with a filename:
yyyyyy.xxx.txt. where yyyyyy are six digits, xxx. as above, and the extension txt is added make your browser quess it is a plain text file, which should be saved to the disk.
Some signals with be put up on the web site. Compute the continuous wavelet coefficients for some set of signals (ex. guitar.wav, noisy,guitar.wav), The output will be a matrix of complex numbers. Plot their absolute values in a colour diagram by the function imagesc().