KTH, Matematik

Svar (med reservation för felräkningar) till Kapitel 4.2-4.3 och 8.1-8.2

4.2.1 (a) Linear; $\mathbb{R}^3 \to \mathbb{R}^2$, (b) Nonlinear; $\mathbb{R}^2 \to \mathbb{R}^3$.

4.2.2 (a) $\begin{pmatrix} 2 & -3 & 1 \\ 3 & 5 & -1 \end{pmatrix}$, (b) $\begin{pmatrix} 7 & 2 & -8 \\ 0 & -1 & 5 \\ 4 & 7 & -1 \end{pmatrix}$, (c) $\begin{pmatrix} -1 & 1 \\ 3 & -2 \\ 5 & -7 \end{pmatrix}$.

4.2.3 $\begin{pmatrix} 3 & 5 & -1 \\ 4 & -1 & 1 \\ 3 & 2 & -1 \end{pmatrix}$; $T(-1,2,4) = (3,-2,-3)$.

4.2.4 (a) $\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$, (c) $\begin{pmatrix} 1 & 2 & 1 \\ 1 & 5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

4.2.5 (b) $\begin{pmatrix} 7 & 2 & -1 & 1 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$.

4.2.6 (b) $\begin{pmatrix} 3 \\ 13 \end{pmatrix}$, (c) $\begin{pmatrix} -2x_1 + x_2 + 4x_3 \\ 3x_1 + 5x_2 + 7x_3 \\ 6x_1 - x_3 \end{pmatrix}$.

4.2.7 (b) $T(2,1,-3) = (0,-2,0)$.

4.2.8 (a) $(-1,-2)$, (b) $(1,2)$, (c) $(2,-1)$.

4.2.13 (a) $\left(-2, \frac{\sqrt{3}-2}{2}, \frac{1+2\sqrt{3}}{2}\right)$.

4.2.15 (a) $\left(-2, \frac{\sqrt{3}+2}{2}, \frac{-1+2\sqrt{3}}{2}\right)$, (b) $(-2\sqrt{2}, 1, 0)$, (c) $(1,2,2)$.

4.2.18 (a) $\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

4.2.21 (a) Yes, (b) No.

4.3.6 (a) The standard matrix for T^{-1} is $\begin{pmatrix} 1 & -2 & 2 \\ 2 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$, and

$$T^{-1}(w_1, w_2, w_3) = (w_1 - 2w_2 + 2w_3, 2w_1 + w_2 + w_3, w_1 + w_2).$$

4.3.12b For a reflection about the y-axis, $T(e_1) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ and $T(e_2) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, thus

$$T = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}.$$

4.3.14 (a) $\begin{pmatrix} 1/5 & 0 & 0 \\ 0 & -1/5 & 0 \\ 0 & 0 & 1/5 \end{pmatrix}$.

4.3.23 (a) $\begin{pmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{pmatrix}$, (b) $\left(\frac{1+5\sqrt{3}}{2}, \frac{\sqrt{3}-5}{2}\right)$.

8.1.1 (1) Let $x = (x_1, x_2), y = (y_1, y_2) \in \mathbb{R}^2$.

$$\begin{aligned} T(x+y) &= T(x_1+y_1, x_2+y_2) = ((x_1+y_1)+2(x_2+y_2), 3(x_1+y_1)-(x_2+y_2)) \\ &= (x_1+2x_2, 3x_1-x_2)+(y_1+2y_2, 3y_1-y_2) = T(x)+T(y). \end{aligned}$$

(2) Let $x = (x_1, x_2) \in \mathbb{R}^2$ and $\lambda \in \mathbb{R}$.

$$\begin{aligned} T(\lambda x) &= T(\lambda x_1, \lambda x_2) = (\lambda x_1 + 2\lambda x_2, 3\lambda x_1 - \lambda x_2) \\ &= \lambda(x_1+2x_2, 3x_1-x_2) = \lambda T(x). \end{aligned}$$

8.1.2 Similar to 8.1.1.

8.1.3 Nonlinear.

8.1.4 Linear.

8.1.9 (a) Linear, (b) Nonlinear.

8.1.16 $(-10, -7, 6)$

8.2.3 (a), (b), (c)

8.2.5 (b)

8.2.7 (a) $\left(\frac{1}{2}, 1\right)$, (b) $\left(\frac{3}{2}, -4, 1, 0\right)$, (c) No basis exists since $\ker(T) = \{0\}$.