Institutionen för matematik
KTH
Chaotic Dynamical Systems, Fall 2010

Homework assignment 2

The exercises are due on October 29, 2010

1. A point p is a *non-wandering* point for a map $f : I \to I$ if for any open interval J containing p, there exists $x \in J$ and $n > 0$ such that $f^n(x) \in J$. Let $NW(f)$ denote the set of non-wandering points for f.

   a) Prove that $NW(f)$ is a closed set and that $\omega(x) \subset NW(f)$ for all $x \in I$.

   b) If $f_a(x) = ax(1-x)$, $a > 2 + \sqrt{5}$, show that $NW(f) = \Lambda$ (where Λ is the cantor set defined in the lecture).

   A point p is *recurrent* if for any open interval J containing p there exists $n > 0$ such that $f^n(p) \in J$.

   c) Give an example of a non-periodic recurrent point for f_a when $a > 2 + \sqrt{5}$.

   d) Give an example of a non-wandering point for f_a which is not recurrent.

2. Exercise 6.1 in the textbook (page 266)

3. Exercise 6.3 in the textbook (page 266).

4. Exercise 6.8 in the textbook (page 266).

5. a) Explain why the rotation $R(x) = x + \omega \pmod 1$ preserves the Lebesgue measure on $[0, 1]$

   Assume now that ω is irrational. b) Prove that for each trigonometric polynomial

   $$P_N(t) = \sum_{k=-N}^{N} a_k e^{2\pi ikt}$$

   we have

   $$\frac{1}{n}\sum_{j=0}^{n-1} P_N(x + j\omega) \to \int_0^1 P_N(t)dt \quad \text{as } n \to \infty.$$

   Hint: It is enough to prove this for the monomials $e^{2\pi kit}$. Explain why.

c) Use Weierstrass approximation theorem to conclude that for continuous functions f and all x

$$\frac{1}{n}\sum_{j=0}^{n-1} f(x+j\omega) \to \int_0^1 f(t)dt \quad \text{as } n \to \infty.$$