SF2724: Topics in mathematics IV: Applied topology
Homework set 1

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The solutions to these exercises are to be handed in no later than Thursday, 31st of March. Please pay attention to the presentation as well as the arguments given in the solutions.

Exercise 1

(a). Consider a map $\pi : D^n_- \coprod D^n_+ \to S^n$ given by the formula:

$$\pi(x) = \begin{cases} (x, \sqrt{1-|x|^2}) & \text{if } x \in D^n_+ \\ (x, -\sqrt{1-|x|^2}) & \text{if } x \in D^n_- \end{cases}$$

Prove that S^n has the quotient topology induced by this map.

(b). Consider a map $\pi : D^n_- \times I \coprod D^n_+ \times I \to S^n \times I$ given by the formula:

$$\pi(x,t) = \begin{cases} ((x, \sqrt{1-|x|^2}), t) & \text{if } x \in D^n_+ \\ ((x, -\sqrt{1-|x|^2}), t) & \text{if } x \in D^n_- \end{cases}$$

Prove that $S^n \times I$ has the quotient topology induced by π.

Exercise 2

Prove Proposition 1.31.1 from the notes. It states that for any map $f : S^{n-1} \to GL(\mathbf{C}^k)$, $\pi_f : E_f \to S^n$ is a complex vector bundle.

Exercise 3

(a). Show that, for any $k \geq 0$, $GL(\mathbf{C}^k)$ is a connected space.

(b). Describe Vect(S^1) and $K(S^1)$.

Exercise 4

Let X be a compact space and $\alpha : S^n \to X$ be a map. Let us denote by $i : X \to X \cup_\alpha D^{n+1}$ the standard inclusion. Recall that $\tilde{K}(X) = \ker(\text{rank} : K(X) \to \mathbf{Z})$. Show that the following is an exact sequence of abelian groups:

$$\tilde{K}(S^n) \xleftarrow{\tilde{K}(\alpha)} \tilde{K}(X) \xleftarrow{\tilde{K}(i)} \tilde{K}(X \cup_\alpha D^{n+1})$$