SF2735/MM8020
Homological algebra and algebraic topology
Exercise set 3
Examinator: Torsten Ekedahl

The solutions to these exercises are to be handed in no later than Thursday, 8th of December. Please pay attention to the presentation as well as the arguments given in the solutions. You are free to ask questions if there is something you don't understand or you get stuck.

Define the *complex projective space*, denoted $\mathbf{CP}^n$, as the set of 1-dimensional sub-vector spaces of $\mathbf{C}^{n+1}$.

Exercise: (**3 points**) Identify $\mathbf{C}^{n+1}$ with $\mathbf{R}^{2n+2}$ so that the function $|z|^2 = |z_0|^2 + \cdots + |z_n|^2$ becomes the usual (square of the) euclidean length function $|z|^2 = |x_0|^2 + |y_0|^2 + \cdots + |x_n|^2 + |y_n|^2$ and let $S^{2n+1} \subseteq \mathbf{C}^{n+1}$ be the unit sphere. Show that the map $\pi\colon S^{2n+1} \to \mathbf{CP}^n$ which takes $z \in S^{2n+1}$ to $\mathbf{C}z$ is surjective and that z and z' map to the same element precisely when there is a $\lambda \in S^1$ such that $z' = \lambda z$.

We then give $\mathbf{CP}^n$ the quotient topology induced by the surjective map $\pi\colon S^{2n+1} \to \mathbf{CP}^n$.

Exercise:

Exercise: i) (**1 point**) Show that $\mathbf{CP}^n$ is compact.

ii) (**3 points**) Show that $\mathbf{CP}^n$ is a $2n$-dimensional manifold.

iii) (**3 points**) Let $\pi\colon S^{2n+1} \to \mathbf{CP}^n$ be the above quotient map. Show that $\mathbf{CP}^{n+1}$ is homeomorphic to $\mathbf{CP}^n \cup_\pi D^{2n+2}$.