Problem session November 22, SF2736, fall 11.

1. Find an element x in the group $\mathcal{S}_4$ of permutations on the set $\{1, 2, 3, 4\}$ such that
$$(1\ 2\ 4\ 3)x(2\ 1\ 3\ 4) = (1\ 4)(2\ 3)\ .$$

2. Show that there is no element φ in the group $\mathcal{S}_5$ of permutations on the set $\{1, 2, 3, 4, 5\}$ such that
$$\varphi^2 = (1\ 2\ 3)(2\ 3\ 4\ 5)\ .$$

3. Show that the following multiplication table is not the multiplication table of a group:

$\circ$	e	a	b	c	d
e	e	a	b	c	d
a	a	b	d	e	c
b	b	e	c	d	a
c	c	d	a	b	e
d	d	c	e	a	b

4. Can the following table be completed to the multiplication table of a group.

$\circ$	e	a	b	c	d
e	e				
a		e			
b					
c					
d					

5. (a) Find the smallest subgroup of $(Z_{18}, +)$ that contains the elements 3 and 7.
   (b) Find the smallest coset of some subgroup of $(Z_{18}, +)$ that contains the elements 3 and 7.

6. (a) For any two subgroups H and K of a group G show that $H \cap K$ is a subgroup of G.
   (b) Find a group G with two distinct non trivial subgroups H and K of G such that $H \cup K$ is a subgroup of G or show that this is not possible.
   (c) The sizes of the subgroups H and K of G are 52 and 151, respectively, find the size of $H \cap K$.

7. Find a non abelian group of size 66.

8. Is the group $(Z_{19} \setminus \{0\},\ \cdot\ )$ a cyclic group.

9. Assume that the two elements a and b in a group G commute, i.e., $ab = ba$. Is it then always true that the order $\sigma(ab)$ of the element ab satisfies
$$\sigma(a, b) = \operatorname{lcm}(\sigma(a), \sigma(b)).$$

10. Show that every subgroup of a cyclic group is cyclic.

11. Show that every group with 55 elements contains at least one element of order 5 and at least one element of order 11.