

To be handed in no later than **April 17**. You may cooperate but you must write your solutions by yourselves. Please write full proofs!

- (1) A function $f : \mathbb{Z}_n \rightarrow \mathbb{Z}_k$ is *periodic* of period r , where $1 \leq r \leq n-1$, if

$$f(m+r) = f(m), \quad \text{for all } m \in \mathbb{Z}_n.$$

Otherwise f is *non-periodic*. For $n, k \geq 1$, let $N(n, k)$ be the number of non-periodic functions $f : \mathbb{Z}_n \rightarrow \mathbb{Z}_k$ and prove

$$N(n, k) = \sum_{d|n} \mu(d, n) k^d,$$

where μ is the Möbius function of the division lattice D_n .

- (2) Exercise 3.42,
- (3) Exercise 3.57,
- (4) Exercise 3.85,
- (5) Exercise 3.90,
- (6) Exercise 3.114a,
- (7) Exercise 3.129.