Formler ur Asmar: PDE, 2:a uppl.: de 3 sista sidorna plus appendix B (10 sidor)

USEFUL FORMULAS

Trigonometric Identities

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\cos(a-b) = \cos a \cos b + \sin a \sin b$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\sin(a-b) = \sin a \cos b - \cos a \sin b$

$$\cos\left(a - \frac{\pi}{2}\right) = \sin a \qquad \sin\left(a + \frac{\pi}{2}\right) = \cos a$$

$$\cos^2 a = \frac{1+\cos 2a}{2} \qquad \sin^2 a = \frac{1-\cos 2a}{2}$$

$$\cos 2a = \cos^2 a - \sin^2 a \qquad \sin 2a = 2\sin a \cos a$$

$$\cos a + \sin a = \sqrt{2}\cos(a - \frac{\pi}{4}) \qquad \cos a + \sin a = \sqrt{2}\sin(a + \frac{\pi}{4})$$

$$\alpha \cos a + \beta \sin a = \sqrt{\alpha^2+\beta^2}\cos(a-b), \text{ where } \cos b = \frac{\alpha}{\sqrt{\alpha^2+\beta^2}} \text{ and } \sin b = \frac{\beta}{\sqrt{\alpha^2+\beta^2}}$$

Complex Numbers

$$z = x + iy \qquad \overline{z} = x - iy \qquad \text{Re}\,(z) = x, \quad \text{Im}\,(z) = y$$

$$z + \overline{z} = 2\,\text{Re}\,(z) = 2x \qquad z - \overline{z} = 2i\,\text{Im}\,(z) = 2iy \qquad |z| = \sqrt{z\cdot\overline{z}} = \sqrt{x^2+y^2}$$

$$|z|^2 = z\cdot\overline{z} = x^2+y^2 \qquad z^2 = x^2 - y^2 + 2ixy \qquad \frac{1}{z} = \frac{\overline{z}}{z\cdot\overline{z}} = \frac{x-iy}{x^2+y^2} \quad (z \neq 0)$$

Triangle Inequality: If z and w are any complex numbers, then

$$|z \pm w| \le |z| + |w| \quad \text{and} \quad \big|\,|z| - |w|\,\big| \le |z \pm w|$$

Euler's Identity and Related Identities: If x is any real number, then

$$e^{ix} = \cos x + i\sin x \qquad e^{-ix} = \cos x - i\sin x \qquad e^{-ix} = \overline{(e^{ix})} \qquad |e^{ix}| = 1$$

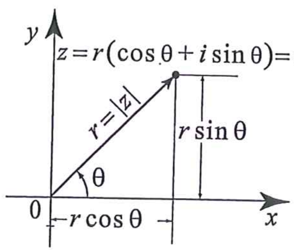
$$\cos x = \frac{e^{ix}+e^{-ix}}{2} \qquad \sin x = \frac{e^{ix}-e^{-ix}}{2i} \qquad (\cos nx + i\sin nx)^n = e^{inx} = \cos nx + i\sin nx$$

Polar Representation of Complex Numbers

$$z = x + iy = re^{i\theta} = r(\cos\theta + i\sin\theta)$$

$$r = |z| = \sqrt{x^2+y^2}$$

$$x = r\cos\theta, \quad y = r\sin\theta$$



Taylor Series Expansions: Let z be a real or complex number. Then

$$e^z = \sum_{n=0}^{\infty}\frac{z^n}{n!} \;(\text{any } z) \qquad \cos z = \sum_{n=0}^{\infty}(-1)^n\frac{z^{2n}}{(2n)!} \;(\text{any } z) \qquad \sin z = \sum_{n=0}^{\infty}(-1)^n\frac{z^{2n+1}}{(2n+1)!} \;(\text{any } z)$$

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n \;(|z|<1) \qquad \cosh z = \sum_{n=0}^{\infty}\frac{z^{2n}}{(2n)!} \;(\text{any } z) \qquad \sinh z = \sum_{n=0}^{\infty}\frac{z^{2n+1}}{(2n+1)!} \;(\text{any } z)$$

Useful Integrals (Take $a \neq 0$ and b to be real, and m and n to be integers.)

Integrals Involving Trigonometric Functions

$$\int \cos\left(ax+b\right)dx = \frac{1}{a}\sin\left(ax+b\right) + C \qquad \int \sin\left(ax+b\right)dx = -\frac{1}{a}\cos\left(ax+b\right) + C$$

$$\int \cos^2\left(ax+b\right)dx = \frac{x}{2} + \frac{1}{4a}\sin\left(2(ax+b)\right) + C$$

$$\int \sin^2\left(ax+b\right)dx = \frac{x}{2} - \frac{1}{4a}\sin\left(2(ax+b)\right) + C$$

$$\int x\cos ax\,dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax + C \qquad \int x\sin ax\,dx = \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax + C$$

$$\int x^2\cos ax\,dx = \frac{2x\cos ax}{a^2} + \frac{a^2x^2-2}{a^3}\sin ax + C$$

$$\int x^2\sin ax\,dx = \frac{2x}{a^2}\sin ax - \frac{a^2x^2-2}{a^3}\cos ax + C$$

$$\int x^m\cos ax\,dx = \frac{x^m\sin ax}{a} - \frac{m}{a}\int x^{m-1}\sin ax\,dx$$

$$\int x^m\sin ax\,dx = -\frac{x^m\cos ax}{a} + \frac{m}{a}\int x^{m-1}\cos ax\,dx$$

$$\int \cos ax\cos bx\,dx = \frac{\sin[(a-b)x]}{2(a-b)} + \frac{\sin[(a+b)x]}{2(a+b)} + C \quad (a^2 \neq b^2)$$

$$\int \sin ax\sin bx\,dx = \frac{\sin[(a-b)x]}{2(a-b)} - \frac{\sin[(a+b)x]}{2(a+b)} + C \quad (a^2 \neq b^2)$$

$$\int \cos ax\sin bx\,dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)} + C \quad (a^2 \neq b^2)$$

Integrals Involving Exponential Functions

$$\int xe^{ax+b}\,dx = \frac{e^{ax+b}}{a^2}\left(ax-1\right) + C \qquad \int x^me^{ax+b}\,dx = \frac{x^me^{ax+b}}{a} - \frac{m}{a}\int x^{m-1}e^{ax+b}\,dx$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\cos bx + b\sin bx\right) + C \qquad (a^2+b^2 \neq 0)$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\sin bx - b\cos bx\right) + C \qquad (a^2+b^2 \neq 0)$$

Identities Involving Bessel Functions (Take $p \geq 0,\ a \neq 0,\ n = 0, 1, \ldots$.)

$$\frac{d}{dx}[J_0(x)] = -J_1(x) \qquad \frac{d}{dx}[x^p J_p(x)] = x^p J_{p-1}(x)$$

$$\frac{d}{dx}[x^{-p} J_p(x)] = -x^{-p} J_{p+1}(x) \qquad x J_p'(x) + p J_p(x) = x J_{p-1}(x)$$

$$x J_p'(x) - p J_p(x) = -x J_{p+1}(x) \qquad J_{p-1}(x) - J_{p+1}(x) = 2\, J_p'(x)$$

$$J_{p-1}(x) + J_{p+1}(x) = \frac{2p}{x} J_p(x) \qquad J_n(x) = \frac{1}{\pi}\int_0^\pi \cos(n\theta - x\sin\theta)\, d\theta$$

$$\int x^{p+1} J_p(x)\, dx = x^{p+1} J_{p+1}(x) + C \qquad \int x^{-p+1} J_p(x)\, dx = -x^{-p+1} J_{p-1}(x) + C$$

$$\int J_1(x)\, dx = -J_0(x) + C \qquad \int x J_0(x)\, dx = x J_1(x) + C$$

$$\int J_{p+1}(x)\, dx = \int J_{p-1}(x)\, dx - 2 J_p(x) \qquad x J_{p+1}(x) + p \int J_{p+1}(x)\, dx = \int x J_p(x)\, dx$$

$$\int J_{2n+1}(x)\, dx = -J_0(x) - 2\sum_{k=1}^{n} J_{2k}(x) + C \qquad \int_0^a x^{p+1} J_p\left(\frac{\alpha}{a}x\right) dx = \frac{a^{p+2}}{\alpha} J_{p+1}(\alpha)$$

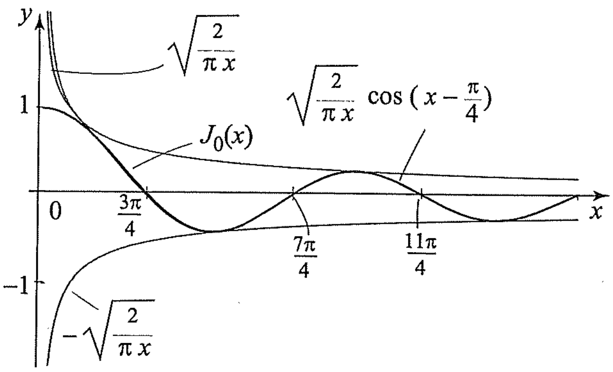
$$\int x J_{2n}(x)\, dx = x J_{2n+1}(x) - 2n J_0(x) - 4n \sum_{k=1}^{n} J_{2k}(x) + C$$

Zeros and Asymptotics of Bessel Functions (Take $n = 0, 1, 2, \ldots$.)

$$J_n(x) \sim \sqrt{\tfrac{2}{\pi x}} \cos\left(x - \tfrac{\pi}{4} - \tfrac{n\pi}{2}\right) + \mathcal{O}\left(\tfrac{1}{x^{3/2}}\right)$$

If α_k = kth positive zero of $J_n(x)$, then for large k

$$\alpha_k \approx \frac{\pi}{4} + \frac{(n+1)\pi}{2} + k\pi$$



Improper Integrals (Take $a \neq 0$.)

$$\int_{-\infty}^{\infty} \frac{\sin ax}{x}\, dx = \frac{\pi}{2}, \text{ if } a > 0;\ -\tfrac{\pi}{2} \text{ if } a < 0. \qquad \int_0^\infty e^{-a^2x^2}\, dx = \frac{\sqrt{\pi}}{2a}$$

$$\int_0^\infty e^{-a^2x^2} \cos bx\, dx = \frac{\sqrt{\pi}}{2|a|} e^{-b^2/(4a^2)}\ (b \neq 0) \qquad \int_0^\infty \frac{\cos wx}{a^2 + x^2}\, dx = \frac{\pi}{2|a|} e^{-|wa|}$$

$$\int_{-\infty}^{\infty} \cos(x^2)\, dx = \int_{-\infty}^{\infty} \sin(x^2)\, dx = \sqrt{\frac{\pi}{2}}$$

Table of Fourier Transforms

	$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \widehat{f}(\omega) e^{ix\omega}\, d\omega$	$\widehat{f}(\omega) = \mathcal{F}(f)(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x}\, dx$
1.	$\begin{cases} 1 & \text{if } \lvert x \rvert < a \\ 0 & \text{if } \lvert x \rvert > a \end{cases}$	$\sqrt{\frac{2}{\pi}} \frac{\sin a\omega}{\omega}$
2.	$\begin{cases} 1 & \text{if } a < x < b \\ 0 & \text{otherwise} \end{cases}$	$\frac{i\left(e^{-i b\omega} - e^{i a\omega}\right)}{\sqrt{2\pi}\omega}$
3.	$\begin{cases} 1 - \frac{\lvert x \rvert}{a} & \text{if } \lvert x \rvert < a \\ 0 & \text{if } \lvert x \rvert > a \end{cases} \quad a > 0$	$2\sqrt{\frac{2}{\pi}} \frac{\sin^2(\frac{a\omega}{2})}{a\omega^2}$
4.	$\begin{cases} x & \text{if } \lvert x \rvert < a \\ 0 & \text{if } \lvert x \rvert > a \end{cases} \quad a > 0$	$i\sqrt{\frac{2}{\pi}} \frac{a\omega\cos(a\omega) - \sin(a\omega)}{\omega^2}$
5.	$\begin{cases} \sin x & \text{if } \lvert x \rvert < \pi \\ 0 & \text{if } \lvert x \rvert > \pi \end{cases}$	$i\sqrt{\frac{2}{\pi}} \frac{\sin(\pi\omega)}{\omega^2 - 1}$
6.	$\begin{cases} \sin(ax) & \text{if } \lvert x \rvert < b \\ 0 & \text{if } \lvert x \rvert > b \end{cases} \quad a, b > 0$	$i\sqrt{\frac{2}{\pi}} \frac{\omega\cos(b\omega)\sin(ab) - a\cos(ab)\sin(b\omega)}{\omega^2 - a^2}$
7.	$\frac{1}{a^2 + x^2},\ a > 0$	$\sqrt{\frac{\pi}{2}} \frac{e^{-a\lvert\omega\rvert}}{a}$
8.	$\frac{x}{a^2 + x^2},\ a > 0$	$-i\sqrt{\frac{\pi}{2}} \operatorname{sgn}\omega\, e^{-a\lvert\omega\rvert}$
9.	$\sqrt{\frac{2}{\pi}} \frac{a}{1 + a^2x^2},\ a > 0$	$e^{-\frac{\lvert\omega\rvert}{a}}$
10.	$\frac{\sin ax}{x},\ a > 0$	$\begin{cases} \sqrt{\frac{\pi}{2}} & \text{if } \lvert\omega\rvert < a \\ \frac{1}{2}\sqrt{\frac{\pi}{2}} & \text{if } \lvert\omega\rvert = a \\ 0 & \text{if } \lvert\omega\rvert > a \end{cases}$
11.	$\frac{4}{\sqrt{2\pi}} \frac{\sin^2(\frac{1}{2}ax)}{ax^2},\ a > 0$	$\begin{cases} 1 - \frac{\lvert\omega\rvert}{a} & \text{if } \lvert\omega\rvert < a \\ 0 & \text{if } \lvert\omega\rvert > a \end{cases}$
12.	$\frac{4}{\sqrt{2\pi}} \frac{\sin^2(ax) - \sin^2(\frac{1}{2}ax)}{ax^2},\ a > 0$	$\begin{cases} 1 & \text{if } \lvert x \rvert < a \\ (-x + 2a)/a & \text{if } a < x < 2a \\ (x + 2a)/a & \text{if } a < x < 2a \\ 0 & \text{if } \lvert x \rvert > 2a \end{cases}$
13.	$e^{-a\lvert x \rvert},\ a > 0$	$\sqrt{\frac{2}{\pi}} \frac{a}{a^2 + \omega^2}$
14.	$\begin{cases} e^{-ax} & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases},\ a > 0$	$\frac{1}{\sqrt{2\pi}} \frac{1}{a + i\omega}$
15.	$\begin{cases} 0 & \text{if } x > 0 \\ e^{ax} & \text{if } x < 0 \end{cases},\ a > 0$	$\frac{1}{\sqrt{2\pi}} \frac{1}{a - i\omega}$
16.	$\lvert x \rvert^n e^{-a\lvert x \rvert},\ a > 0,\ n > 0$	$\frac{\Gamma(n+1)}{\sqrt{2\pi}} \left(\frac{1}{(a - i\omega)^{1+n}} + \frac{1}{(a + i\omega)^{1+n}} \right)$

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Table of Fourier Transforms (continued)

$f(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\widehat{f}(\omega)e^{ix\omega}\,d\omega$	$\widehat{f}(\omega)=\mathcal{F}(f)(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(x)e^{-i\omega x}\,dx$
17. $e^{-\frac{a}{2}x^2}$, $a>0$	$\frac{1}{\sqrt{a}}e^{-\frac{\omega^2}{2a}}$
18. e^{-ax^2}, $a>0$	$\frac{1}{\sqrt{2a}}e^{-\frac{\omega^2}{4a}}$
19. $xe^{-\frac{a}{2}x^2}$, $a>0$	$\frac{-i\omega}{a^{3/2}}e^{-\frac{\omega^2}{2a}}$
20. $x^2e^{-\frac{a}{2}x^2}$, $a>0$	$\frac{a-\omega^2}{a^{5/2}}e^{-\frac{\omega^2}{2a}}$
21. $x^3e^{-\frac{a}{2}x^2}$, $a>0$	$\frac{-i\omega(3a-\omega^2)}{a^{7/2}}e^{-\frac{\omega^2}{2a}}$
22. $e^{-\frac{x^2}{2}}H_n(x)$, H_n, nth Hermite polynomial	$(-1)^n i^n e^{\frac{-\omega^2}{2}}H_n(\omega)$
23. $J_0(x)$, Bessel function of order 0	$\begin{cases}\sqrt{\frac{2}{\pi}}\frac{1}{\sqrt{1-\omega^2}} & \text{if } \|\omega\|<1\\ 0 & \text{if } \|\omega\|>1\end{cases}$
24. $J_n(x)$, Bessel function of order $n\geq 0$	$\begin{cases}\sqrt{\frac{2}{\pi}}\frac{(-i)^n}{\sqrt{1-\omega^2}}T_n(\omega) & \text{if } \|\omega\|<1\\ 0 & \text{if } \|\omega\|>1\end{cases}$ T_n, Chebyshev polynomial of degree n.

Special Transforms

25. $\mathcal{F}(\delta_0(x))(\omega)=\frac{1}{\sqrt{2\pi}}$	**27.** $\mathcal{F}(\sqrt{\frac{2}{\pi}}\frac{1}{x})(\omega)=-i\,\mathrm{sgn}\,\omega$
26 $\mathcal{F}(\delta_0(x-a))(\omega)=\frac{1}{\sqrt{2\pi}}e^{-ia\omega}$	**28.** $\mathcal{F}(e^{iax})(\omega)=\sqrt{2\pi}\delta_0(\omega-a)$

Operational Properties

29. $\mathcal{F}(af+bg)(\omega)=a\mathcal{F}(f)+b\mathcal{F}(g)$	**36.** $\mathcal{F}(f\,g)(\omega)=\mathcal{F}(f)*\mathcal{F}(g)(\omega)$
30. $\mathcal{F}(f')(\omega)=i\omega\mathcal{F}(f)(\omega)$	**37.** $\mathcal{F}(f(x-a))(\omega)=e^{-ia\omega}\mathcal{F}(f)(\omega)$
31. $\mathcal{F}(f'')(\omega)=-\omega^2\mathcal{F}(f)(\omega)$	**38.** $\mathcal{F}(e^{iax}f(x))(\omega)=\mathcal{F}(f)(\omega-a)$
32. $\mathcal{F}(f^{(n)})(\omega)=(i\omega)^n\mathcal{F}(f)(\omega)$	**39.** $\mathcal{F}(\cos(ax)f(x))(\omega)=\frac{\mathcal{F}(f)(\omega-a)+\mathcal{F}(f)(\omega+a)}{2}$
33. $\mathcal{F}(xf(x))(\omega)=i\frac{d}{d\omega}\mathcal{F}(f)(\omega)$	**40.** $\mathcal{F}(\sin(ax)f(x))(\omega)=\frac{\mathcal{F}(f)(\omega-a)-\mathcal{F}(f)(\omega+a)}{2i}$
34. $\mathcal{F}(x^nf(x))(\omega)=i^n\frac{d^n}{d\omega^n}\mathcal{F}(f)(\omega)$	**41.** $\mathcal{F}(f(ax))(\omega)=\frac{1}{\|a\|}\mathcal{F}(f)(\frac{\omega}{a})$, $a\neq 0$
35. $\mathcal{F}(f*g)(\omega)=\mathcal{F}(f)(\omega)\mathcal{F}(g)(\omega)$	**42.** $f(x)=\mathcal{F}(\widehat{f})(-x)$, $\mathcal{F}(\mathcal{F}(f))=f(-x)$

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Table of Fourier Cosine Transforms

$f(x)=\sqrt{\frac{2}{\pi}}\int_0^\infty \mathcal{F}_c(f)(\omega)\cos\omega x\,d\omega,\quad 0<x<\infty$	$\mathcal{F}_c(f)(\omega)=\widehat{f}_c(\omega)=\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos\omega x\,dx,\quad 0\le\omega<\infty$
1. $\begin{cases}1 & \text{if } 0<x<a\\ 0 & \text{otherwise}\end{cases}$	$\sqrt{\frac{2}{\pi}}\frac{\sin a\omega}{\omega}$
2. $e^{-ax},\quad a>0$	$\sqrt{\frac{2}{\pi}}\frac{a}{a^2+\omega^2}$
3. $x\,e^{-ax},\quad a>0$	$\sqrt{\frac{2}{\pi}}\frac{a^2-\omega^2}{(a^2+\omega^2)^2}$
4. $e^{-a x^2/2},\quad a>0$	$\frac{1}{\sqrt{a}}e^{-\omega^2/2a}$
5. $\cos ax\,e^{-ax},\quad a>0$	$\sqrt{\frac{2}{\pi}}\frac{a\omega^2+2a^3}{4a^4+\omega^4}$
6. $\sin ax\,e^{-ax},\quad a>0$	$\sqrt{\frac{2}{\pi}}\frac{2a^3-a\omega^2}{4a^4+\omega^4}$
7. $\frac{a}{a^2+x^2},\quad a>0$	$\sqrt{\frac{\pi}{2}}e^{-a\omega}$
8. $x^p,\quad 0<p<1$	$\sqrt{\frac{2}{\pi}}\frac{\Gamma(p)\cos(p\omega/2)}{\omega^p}$
9. $\begin{cases}\cos x & \text{if } 0<x<a\\ 0 & \text{otherwise}\end{cases}$	$\frac{1}{\sqrt{2\pi}}\left[\frac{\sin a(1-\omega)}{1-\omega}+\frac{\sin a(1+\omega)}{1+\omega}\right]$
Operational Properties	
10. $\alpha f(x)+\beta g(x)$	$\alpha\mathcal{F}_c(f)(\omega)+\beta\mathcal{F}_c(g)(\omega)$
11. $f(ax),\quad a>0$	$\frac{1}{a}\widehat{f}_c(\frac{\omega}{a})$
12. $f'(x)$	$\omega\widehat{f}_s(\omega)-\sqrt{\frac{2}{\pi}}\,f(0)$
13. $f''(x)$	$-\omega^2\widehat{f}_c(\omega)-\sqrt{\frac{2}{\pi}}\,f'(0)$
14. $xf(x)$	$\left[\widehat{f}_s\right]'(\omega)$
15. $\mathcal{F}_c(\mathcal{F}_c f)$	f

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Table of Fourier Sine Transforms

$f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty \mathcal{F}_s(f)(\omega) \sin \omega x \, d\omega$, $0 < x < \infty$	$\mathcal{F}_s(f)(\omega) = \widehat{f}_s(\omega) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin \omega x \, dx$, $0 < \omega < \infty$
1. $\begin{cases} 1 & \text{if } 0 < x < a \\ 0 & \text{otherwise} \end{cases}$	$\sqrt{\frac{2}{\pi}} \frac{1-\cos a\omega}{\omega}$
2. e^{-ax}, $a > 0$	$\sqrt{\frac{2}{\pi}} \frac{\omega}{a^2+\omega^2}$
3. $x e^{-ax}$, $a > 0$	$\sqrt{\frac{2}{\pi}} \frac{2a\omega}{(a^2+\omega^2)^2}$
4. $\frac{e^{-ax}}{x}$, $a > 0$	$\sqrt{\frac{2}{\pi}} \tan^{-1} \frac{\omega}{a}$
5. $\frac{1}{2} x e^{-a x^2}$, $a > 0$	$\frac{\omega}{a^{3/2}} e^{-\omega^2/2a}$
6. $\cos ax \, e^{-ax}$, $a > 0$	$\sqrt{\frac{2}{\pi}} \frac{\omega^3}{4a^4+\omega^4}$
7. $\sin ax \, e^{-ax}$, $a > 0$	$\sqrt{\frac{2}{\pi}} \frac{2a^2\omega}{4a^4+\omega^4}$
8. $\frac{x}{a^2+x^2}$, $a > 0$	$\sqrt{\frac{\pi}{2}} e^{-a\omega}$
9. x^{p-1}, $0 < p < 1$	$\sqrt{\frac{2}{\pi}} \frac{\Gamma(p) \cos(\pi p/2)}{\omega^p}$
10. $\begin{cases} \sin x & \text{if } 0 < x < a \\ 0 & \text{otherwise} \end{cases}$	$\frac{1}{\sqrt{2\pi}} \left[\frac{\sin a(1-\omega)}{1-\omega} - \frac{\sin a(1+\omega)}{1+\omega} \right]$

Operational Properties

11. $\alpha f(x) + \beta g(x)$	$\alpha \mathcal{F}_s(f)(\omega) + \beta \mathcal{F}_s(g)(\omega)$
12. $f(ax)$, $a > 0$	$\frac{1}{a} \widehat{f}_s(\frac{\omega}{a})$
13. $f'(x)$	$-\omega \widehat{f}_c(\omega)$
14. $f''(x)$	$-\omega^2 \widehat{f}_s(\omega) + \sqrt{\frac{2}{\pi}} \omega f(0)$
15. $x f(x)$	$-\left[\widehat{f}_c\right]'(\omega)$
16. $\mathcal{F}_s(\mathcal{F}_s f)$	f

Table of Laplace Transforms

	$f(t), \quad t \geq 0$	$F(s) = \mathcal{L}(f)(s) = \int_0^\infty f(t)e^{-st}\,dt,$
1.	1	$\frac{1}{s}, \quad s > 0$
2.	$t^n, \quad n = 1, 2, \ldots$	$\frac{n!}{s^{n+1}}, \quad s > 0$
3.	$t^a \quad (a > -1)$	$\frac{\Gamma(a+1)}{s^{a+1}}, \quad s > 0$
4.	e^{at}	$\frac{1}{s-a}, \quad s > a$
5.	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
6.	$\frac{e^{at}-e^{bt}}{a-b}$	$\frac{1}{(s-a)(s-b)}, \quad s > \max(a, b)$
7.	$\frac{ae^{at}-be^{bt}}{a-b}$	$\frac{s}{(s-a)(s-b)}, \quad s > \max(a, b)$
8.	$\sin kt$	$\frac{k}{s^2+k^2}, \quad s > 0$
9.	$\cos kt$	$\frac{s}{s^2+k^2}, \quad s > 0$
10.	$e^{at} \sin kt$	$\frac{k}{(s-a)^2+k^2}, \quad s > a$
11.	$e^{at} \cos kt$	$\frac{s-a}{(s-a)^2+k^2}, \quad s > a$
12.	$t \sin kt$	$\frac{2ks}{(s^2+k^2)^2}, \quad s > 0$
13.	$t \cos kt$	$\frac{s^2-k^2}{(s^2+k^2)^2}, \quad s > 0$
14.	$\frac{1}{2a^3}(\sin at - at \cos at)$	$\frac{1}{(s^2+a^2)^2}, \quad s > 0$
15.	$\sinh kt$	$\frac{k}{s^2-k^2}, \quad s > \lvert k\rvert$
16.	$\cosh kt$	$\frac{s}{s^2-k^2}, \quad s > \lvert k\rvert$
17.	$e^{at} \sinh kt$	$\frac{k}{(s-a)^2-k^2} \quad s > a + \lvert k\rvert$
18.	$e^{at} \cosh kt$	$\frac{s-a}{(s-a)^2-k^2} \quad s > a + \lvert k\rvert$
19.	$t \sinh kt$	$\frac{2ks}{(s^2-k^2)^2}, \quad s > \lvert k\rvert$
20.	$t \cosh kt$	$\frac{s^2+k^2}{(s^2-k^2)^2}, \quad s > \lvert k\rvert$
21.	$\frac{1}{2k^3}(kt \cosh kt - \sinh kt)$	$\frac{1}{(s^2-k^2)^2}, \quad s > \lvert k\rvert$

Table of Laplace Transforms (continued)

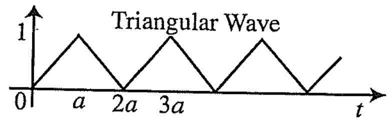
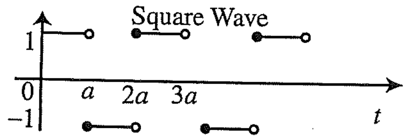
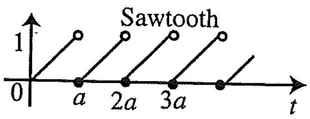
	$f(t), \quad t \geq 0$	$F(s) = \mathcal{L}(f)(s) = \int_0^{\infty} f(t)e^{-st}\,dt,$
22.	$\delta_0(t-t_0), \quad t_0 \geq 0$	$e^{-t_0 s}, \quad s > 0$
23.	$\mathcal{U}_0(t-a) = \begin{cases} 0 & \text{if } t < a \\ 1 & \text{if } t \geq a \end{cases} \quad (a > 0)$	$\frac{e^{-as}}{s}, \quad s > 0$
24.	$f(t+T) = f(t) \quad (T > 0)$	$\frac{1}{1-e^{-Ts}} \int_0^T e^{-st} f(t)\,dt$
25.	$f(t+T) = -f(t) \quad (T > 0)$	$\frac{1}{1+e^{-Ts}} \int_0^T e^{-st} f(t)\,dt$
26.		$\frac{1}{as^2}\left[\frac{1-e^{-as}}{1+e^{-as}}\right] = \frac{1}{as^2}\tanh\left(\frac{as}{2}\right), \quad s > 0$
27.		$\frac{1}{s}\left[\frac{1-e^{-as}}{1+e^{-as}}\right] = \frac{1}{s}\tanh\left(\frac{as}{2}\right), \quad s > 0$
28.		$\frac{1}{as^2} - \frac{e^{-as}}{s(1-e^{-as})}, \quad s > 0$
29.	$\frac{\sin at}{t}$	$\tan^{-1}\left(\frac{a}{s}\right), \quad s > 0$
30.	$J_0(at)$	$\frac{1}{\sqrt{s^2+a^2}}, \quad s > 0$
31.	$J_0(a\sqrt{t})$	$\frac{e^{-a^2/4s}}{s}, \quad s > 0$
32.	$t^p J_p(at) \quad (p > -\frac{1}{2})$	$\frac{2^p a^p \Gamma(p+\frac{1}{2})}{\sqrt{\pi}(s^2+a^2)^{p+\frac{1}{2}}}, \quad s > 0$
33.	$\frac{\sqrt{\pi}}{\Gamma(k)}\left(\frac{t}{2a}\right)^{k-\frac{1}{2}} J_{k-\frac{1}{2}}(at) \quad (k > 0)$	$\frac{1}{(s^2+a^2)^k}, \quad s > 0$
34.	$\frac{\sqrt{\pi}}{\Gamma(k)} a \left(\frac{t}{2a}\right)^{k-\frac{1}{2}} J_{k-\frac{3}{2}}(at) \quad (k > \frac{1}{2})$	$\frac{s}{(s^2+a^2)^k}, \quad s > 0$
35.	$2\sum_{m=1}^{n} \binom{2n-m-1}{n-1} \frac{t^{m-1}\cos(at - \frac{m\pi}{2})}{(2a)^{2n-m}(m-1)!}$ (n an integer ≥ 1)	$\frac{1}{(s^2+a^2)^n}, \quad s > 0$
36.	$\frac{1}{(n-1)!}\sum_{m=1}^{n-1} \frac{(2n-m-3)!}{(m-1)!(n-m-1)!} \frac{t^m \cos(at - \frac{m\pi}{2})}{(2a)^{2n-m-2}}$ (n an integer ≥ 2)	$\frac{s}{(s^2+a^2)^n}, \quad s > 0$

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Table of Laplace Transforms (continued)

	$f(t), \quad t \geq 0$	$F(s) = \mathcal{L}(f)(s) = \int_0^\infty f(t)e^{-st}\,dt,$
37.	$\operatorname{erf}(at) \quad (a > 0)$	$\frac{1}{s}\, e^{s^2/4a^2} \operatorname{erfc}\left(\frac{s}{2a}\right), \quad s > 0$
38.	$\operatorname{erf}(a\sqrt{t})$	$\frac{a}{s\sqrt{s+a^2}}, \quad s > 0$
39.	$e^{-a^2t^2} \quad (a > 0)$	$\frac{\sqrt{\pi}}{2a} e^{s^2/4a^2} \operatorname{erfc}(\frac{s}{2a}), \quad s > 0$
40.	$\frac{1}{\sqrt{\pi t}} e^{-a^2/4t} \quad (a \geq 0)$	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}}, \quad s > 0$
41.	$\frac{a}{2\sqrt{\pi} t^{3/2}} e^{-a^2/4t} \quad (a > 0)$	$e^{-a\sqrt{s}}, \quad s > 0$
42.	$\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right) \quad (a \geq 0)$	$\frac{1}{s}\, e^{-a\sqrt{s}}, \quad s > 0$
	Operational Properties	
43.	$\alpha f(t) + \beta g(t)$	$\alpha F(s) + \beta G(s)$
44.	$f'(t)$	$sF(s) - f(0)$
45.	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
46.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \ldots - f^{(n-1)}(0)$
47.	$-tf(t)$	$F'(s)$
48.	$t^nf(t)$	$(-1)^nF^{(n)}(s)$
49.	$\int_0^t f(\tau)\,d\tau$	$\frac{1}{s}\, F(s), \quad s > 0$
50.	$\int_0^t \int_0^\tau f(\rho)\,d\rho d\tau$	$\frac{1}{s^2}\, F(s), \quad s > 0$
51.	$\frac{f(t)}{t}$	$\int_s^\infty F(u)\,du$
52.	$\frac{f(t)}{t^2}$	$\int_s^\infty \int_\sigma^\infty F(u)\,du d\sigma$
53.	$\mathcal{U}(t-a)f(t-a) \quad (a > 0)$	$e^{-as}F(s)$
54.	$e^{at}f(t)$	$F(s-a)$
55.	$f(ct) \quad (c > 0)$	$\frac{1}{c}F(\frac{s}{c})$
56.	$f * g(t) = \int_0^t f(\tau)g(t-\tau)\,d\tau$	$F(s)G(s)$