1 $T = 3x^2 - 2xyz + y^2z$

$\operatorname{grad} T = (6x - 2yz, -2xz + 2yz, -2xy + y^2)$

I punkten $(1,2,3)$:

$\operatorname{grad} T = (-6, 6, 0)$

$\frac{1}{3}(2,-1,-2) \cdot \operatorname{grad} T = \frac{1}{3} \cdot (-18) = -6$

hastigheten $= 4 \cdot (-6) = -24$

② $\bar{F} = (2xyz,\ x^2z+1,\ x^2y)$

a)

$$\operatorname{rot}\bar{F} = \det\begin{pmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xyz & x^2z+1 & x^2y \end{pmatrix} =$$

$$= (x^2 - x^2,\ 2xy - 2xy,\ 2xz - 2xz) = 0.$$

Potential ϕ ur

$$\begin{cases} \frac{\partial \phi}{\partial x} = 2xyz & (1) \\ \frac{\partial \phi}{\partial y} = x^2z+1 & (2) \\ \frac{\partial \phi}{\partial z} = x^2y & (3) \end{cases}$$

$$(1) \Leftrightarrow \phi = x^2yz + f(y,z)$$

$$\frac{\partial \phi}{\partial y} = x^2z + \frac{\partial f}{\partial y}$$

$$(2) \Leftrightarrow \frac{\partial f}{\partial y} = 1 \Leftrightarrow f = y + g(z)$$

$$\phi = x^2yz + y + g(z)$$

$$\frac{\partial \phi}{\partial z} = x^2y + \frac{\partial g}{\partial z}$$

$$(3) \Leftrightarrow \frac{\partial g}{\partial z} = 0 \Leftrightarrow g = C_1$$

Alltså: $\phi = x^2yz + y + C_1$

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b) $$\int_L \bar{F} \cdot d\bar{r} = \phi(1,2,3) - \phi(0,0,0) =$$

$$= 8.$$

Direkt beräkning:

$\bar{r}(t) = (t, 2t, 3t)$, $0 \le t \le 1$.

$$\int_L \bar{F} \cdot d\bar{r} = \int_0^1 (2 \cdot t \cdot 2t \cdot 3t, t^2 \cdot 3t + 1, t^2 \cdot 2t) \cdot$$

$$\cdot (1, 2, 3)\, dt$$

$$= \int_0^1 (12t^3 + 6t^3 + 2 + 6t^3)\, dt =$$

$$= \int_0^1 (24t^3 + 2)\, dt = \frac{24}{4} + 2 = 8.$$

③ $\bar{F} = \left(x^2 + y^2 z,\ 2xz,\ 3z^4\right)$

$$\text{rot}\,\bar{F} = \det\begin{pmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2+y^2z & 2xz & 3z^4 \end{pmatrix}$$

$$= \left(-2x,\ y^2,\ 2z - 2yz\right).$$

$S:\ \bar{r}(x,y) = (x, y, x^2+y^2), \quad x^2+y^2 \le 1$

$$\frac{\partial \bar{r}}{\partial x} \times \frac{\partial r}{\partial y} = \det\begin{pmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ 1 & 0 & 2x \\ 0 & 1 & 2y \end{pmatrix} =$$

$$= (-2x, -2y, \underbrace{1}_{\text{rätt tecken}})$$

$$\iint_S \operatorname{rot} \bar{F} \cdot \hat{n} \, dS =$$

$$= + \iint_{x^2+y^2 \le 1} \left(-2x, y^2, 2\overset{x^2+y^2}{z} - 2y\overset{x^2+y^2}{z}\right) \cdot (-2x, -2y, 1) \, dx \, dy$$

$$= \iint_{x^2+y^2 \le 1} \left(4x^2 - 2y^3 + 2x^2 + 2y^2 - 2y^3 - 2x^2 y\right) dx \, dy$$

$$= \left\{ \begin{array}{ll} x = r\cos\varphi & 0 \le r \le 1 \\ y = r\sin\varphi & 0 \le \varphi < 2\pi \end{array} \right.$$

$$= \int\int \Big(4r^2\cos^2\varphi - 2r^3\sin^3\varphi + 2r^2\cos^2\varphi +$$

$$+ 2r^2\sin^2\varphi - 2r^3\sin^3\varphi - 2r^3\cos^2\varphi \sin\varphi\Big) r \, dr \, d\varphi$$

$$= \int\int \left(4r^2\cos^2\varphi + 2r^2\right) r \, dr \, d\varphi = 2\pi$$

Med Stokes sats:

$$\iint_S \operatorname{rot} \bar{F} \cdot \hat{n}\, dS = \int_{\partial S} \bar{F} \cdot d\bar{r} =$$

$$= \left\{ \partial S : \bar{r}(t) = (\cos t, \sin t, 1) \right\}$$

$$= \int_0^{2\pi} \Big(\underbrace{\cos^2 t + \sin^2 t}_{1}, 2\cos t, 3 \Big) \cdot (-\sin t, \cos t, 0)\, dt$$

$$= \int_0^{2\pi} \Big(\underbrace{-\cos^2 t \cdot \sin t - \sin^3 t}_{= -\sin t} + 2\cos^2 t \Big)\, dt$$

$$= 2\pi.$$