KTH Teknikvetenskap

SF2729 Groups and Rings
Midterm Exam
Tuesday, March 15, 2011

Time: 09.00-11.00
Allowed aids: none
Examiner: Mats Boij

This midterm exam corresponds to Part I (groups part) of the final exam and the final grade will be based on the maximum of the results on this part in the midterm exam and in the final exam.

Each problem can give up to 6 points. In the first problem, you are guaranteed a minimum given by the result of the homework assignment. If you have at least 2 points from HW, you cannot get anything from Part a) of Problem 1, if you have at least 4 points from HW you cannot get anything from Part a) or Part b) of Problem 1.

The minimum requirements for the various grades are according to the following table:

Grade	**A**	**B**	**C**	**D**	**E**
Total credit	30	27	24	21	18
From Part I	13	12	11	9	8
From Part II	13	12	11	9	8

Present your solutions to the problems in a way such that arguments and calculations are easy to follow.

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(1) a) Show that there are exactly two non-isomorphic group structures on a set of four elements directly from basic properties of groups. **(2)**

b) Draw the Cayley digraph of the symmetric group S_3 using the generators $(1\,2\,3)$ and $(2\,3)$. **(2)**

c) The group of upper-triangular invertible matrices of the field $\mathbb{Z}_3$ with three elements has order 12 as well as the dihedral group D_6 given by the symmetries of a regular hexagon. Show that they are isomorphic. (Recall that the field $\mathbb{Z}_3$ can be seen as the integers modulo 3, i.e., with the usual addition and multiplication of residue classes.) **(2)**

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(2) Let $G = \mathrm{Sl}_2(\mathbb{Z})$ be the group of integer matrices of size 2×2 with determinant one.

a) Show that G acts on $\mathbb{Z}^2$ seen as 1×2-matrices by matrix multiplication by the inverse on the right, i.e., by $A.\begin{pmatrix} m & n \end{pmatrix} = \begin{pmatrix} m & n \end{pmatrix} A^{-1}$. **(2)**

b) Determine the stabilizer[1], G_x, where $x = \begin{pmatrix} 1 & 2 \end{pmatrix}$. **(4)**

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(3) a) Let H be a normal subgroup of a group G. Give the definition of the factor group G/H and prove that this is a well-defined group. **(2)**

b) Let G be the group given by the generators a and b with the relations $a^3 = e$, $b^3 = e$ and $abab = e$, i.e., the factor group of the free group $F[a, b]$ by the smallest normal subgroup containing $\{a^3, b^3, abab\}$. Show that G is isomorphic to the alternating group A_4. **(4)**

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[1] called *isotropy subgroup* in the text-book.