SF2729 Groups and Rings
Problem set 1

due: Tuesday Nov 12 in class.

Write clear, clean, brief, and complete solutions and use whole sentences. Solutions without proper reasoning score worse. You can submit hand-written or typed solutions and turn them in in class or send them by email to `tilmanb@kth.se`. I will not accept late homework except under extraordinary circumstances that you need to discuss with me before the deadline.

Problem 1. Let d be a non-negative integer and put

$$P = \{(x, y) \in \mathbf{Z} \times \mathbf{Z} \mid x^2 - dy^2 = 1\}.$$

Let (x_0, y_0) and (x_1, y_1) be elements in P. Consider the following operation:

$$(x_0, y_0) \cdot (x_1, y_1) = (x_0x_1 + dy_0y_1, x_0y_1 + x_1y_0).$$

Show that $\cdot$ is a binary operation on P and that P endowed with this operation is a group.

Problem 2. Complete the following table so that it is the multiplication table of a group. There is only one possibility. You do not need to write down your reasoning in this exercise.

$\cdot$	e	p	q	r	s	t
e	e	p	q	r	s	t
p	p	q	e	s		
q	q					
r	r			e		p
s	s					
t	t					

Problem 3. Let H be a chess board (with black and white fields) which is infinite in all directions. Let G be the group of rigid-motion symmetries of H (no reflections). Show that every element is either a rotation or a composition of two rotations in G.