SF2729 Groups and Rings
Problem set 10

due: Monday Feb 17 in class.

Write clear, clean, brief, and complete solutions and use whole sentences. Solutions without proper reasoning score worse. You can submit hand-written or typed solutions and turn them in in class or send them by email to `tilmanb@kth.se`. I will not accept late homework except under extraordinary circumstances that you need to discuss with me before the deadline.

Problem 1. Show that the ring $\mathcal{O}$ of integers in $\mathbf{Q}[\sqrt{-2}]$ is a Euclidean ring. Compute a greatest common divisor d of $a = 3$ and $b = -1 + 5\sqrt{-2}$ in $\mathcal{O}$ and find x, $y \in \mathcal{O}$ such that $d = ax + by$.

Problem 2. Show that $\mathbf{Z}[i]/(2 + 5i)$ is a finite field. How many elements does it have?

Problem 3. Let $p(x) = x^6 + 4x^5 + 6x^4 + 7x^3 + 6x^2 + 3x + 1$. Decompose $p(x)$ into irreducible polynomials in each of the rings $\mathbf{F}_2[x]$, $\mathbf{F}_3[x]$, and $\mathbf{F}_5[x]$.