SF2729 Groups and Rings
Problem set 11

due: Monday Feb 24 in class.

Write clear, clean, brief, and complete solutions and use whole sentences. Solutions without proper reasoning score worse. You can submit hand-written or typed solutions and turn them in in class or send them by email to `tilmanb@kth.se`. I will not accept late homework except under extraordinary circumstances that you need to discuss with me before the deadline.

Problem 1. Let p be a prime number with $p \equiv 1 \pmod 4$. We have shown that there exist $a, b \in \mathbf{Z}$ such that $p = a^2 + b^2$. Show that a and b are unique up to order and signs.

Problem 2. Let R be an integral domain and $p(x) = a_n x^n + \cdots + a_1 x + a_0 \in R[x]$ be indecomposable. Show that then also the polynomial $q(x) = a_0 x^n + \cdots + a_{n-1} x + a_n$ is indecomposable.

Problem 3. Show that the polynomial $x^4 + 4x^3 + 6x^2 + 9x + 11$ is indecomposable in $\mathbf{Z}[x]$.