SF2729 Groups and Rings
Problem set 2

due: Tuesday Nov 21 in class.

Write clear, clean, brief, and complete solutions and use whole sentences. Solutions without proper reasoning score worse. You can submit hand-written or typed solutions and turn them in in class or send them by email to `tilmanb@kth.se`. I will not accept late homework except under extraordinary circumstances that you need to discuss with me before the deadline.

Problem 1. Consider the symmetric group S_8 of permutations of $\{1, \ldots, 8\}$.
(1) Compute the product $\sigma = (14238)(274)(35871)(75)$ in cycle notation.
(2) What is the order of σ?
(3) Find an element of S_8 with a larger order than σ.

Problem 2. Let $E = \left(\begin{smallmatrix} 1 & 1 \\ 0 & 1 \end{smallmatrix}\right)$ and define G to be the set of all matrices $M \in \mathrm{GL}_2(\mathbf{R})$ that commute with E, i. e. such that $EM = ME$. Show that G is an abelian group under matrix multiplication.

Problem 3. Show that the symmetric group S_n has a presentation of the form

$$S_n = \langle a_1, \ldots, a_{n-1} \mid a_i^2 = \mathrm{id}, \ a_i a_j = a_j a_i \text{ for } |i - j| > 1, \ (a_i a_{i+1})^3 = \mathrm{id} \rangle$$

Hint: Bubble sort.