SF2729 Groups and Rings
Problem set 3

due: Tuesday Nov 26 in class.

Write clear, clean, brief, and complete solutions and use whole sentences. Solutions without proper reasoning score worse. You can submit hand-written or typed solutions and turn them in in class or send them by email to `tilmanb@kth.se`. I will not accept late homework except under extraordinary circumstances that you need to discuss with me before the deadline.

Problem 1. Let $G = \{g_1, \ldots, g_n\}$ be a finite group of order n. For any $g \in G$, we thus have

$$g \cdot g_i = g_j$$

for some $j = \phi_g(i)$. We can think of ϕ_g as a permutation of $\{1, \ldots, n\}$. Show that the map

$$G \to S_n; \quad g \mapsto \phi_g$$

is an injective group homomorphism.

Problem 2. Determine all subgroups of the dihedral group D_{10} and draw their subgroup lattice. Give short (one-sentence) arguments why your groups are subgroups and why there are no other subgroups.

Problem 3. Under which condition on $m, n \geq 1$ is the group $\mathbf{Z}/m\mathbf{Z} \times \mathbf{Z}/n\mathbf{Z}$ cyclic?