SF2729 Groups and Rings
Problem set 4

due: Wednesday Dec 4 in class.

Write clear, clean, brief, and complete solutions and use whole sentences. Solutions without proper reasoning score worse. You can submit hand-written or typed solutions and turn them in in class or send them by email to `tilmanb@kth.se`. I will not accept late homework except under extraordinary circumstances that you need to discuss with me before the deadline.

Problem 1. Show that the group
$$(\mathbf{Z}/9\mathbf{Z})^{\times} = \{\bar{a} \in \mathbf{Z}/9\mathbf{Z} \mid \gcd(a, 9) = 1\}$$
is cyclic. Which elements of this group are generators?

Problem 2. Let $H < G$ be a subgroup and
$$\overline{H} = \bigcap \left\{ K \trianglelefteq G \mid H \subseteq K \right\}$$
Show that (a) $\overline{H}$ is a normal subgroup of G, and (b) that it is the smallest normal subgroup of G containing H in the following sense: if N is any normal subgroup of G containing H then $\overline{H} \leq N$.

Problem 3. Show that $\mathrm{SL}_n(\mathbf{R})$ is a normal subgroup of $\mathrm{GL}_n(\mathbf{R})$ and that the quotient group $\mathrm{GL}_n(\mathbf{R})/\mathrm{SL}_n(\mathbf{R})$ is isomorphic to $\mathbf{R}^{\times}$.