

# SF2729 Groups and Rings

## Problem set 5

due: Wednesday Dec 11 in class.

Write clear, clean, brief, and complete solutions and use whole sentences. Solutions without proper reasoning score worse. You can submit hand-written or typed solutions and turn them in in class or send them by email to `tilmanb@kth.se`. I will not accept late homework except under extraordinary circumstances that you need to discuss with me before the deadline.

**Problem 1.** Let  $G$  be any group and define  $G'$  to be the subgroup generated by commutators, i. e. elements of the form

$$[x, y] := xyx^{-1}y^{-1}; \quad x, y \in G.$$

Show that  $G'$  is a normal subgroup of  $G$  and that  $G/G'$  is abelian.

**Problem 2.** Let  $G$  be the symmetry group of a cube (it has order 48). Compute the stabilizers of

- (1) one of the eight corners;
- (2) the midpoint of one of the twelve edges;
- (3) the midpoint of one of the six faces.

**Problem 3.** Show that every proper subgroup of a group  $G$  of order 91 is cyclic. Show that  $G$  is either abelian or  $Z(G) = \{1\}$ .