SF2729 Groups and Rings
Problem set 6

due: Wednesday Dec 19 in class.

Write clear, clean, brief, and complete solutions and use whole sentences. Solutions without proper reasoning score worse. You can submit hand-written or typed solutions and turn them in in class or send them by email to `tilmanb@kth.se`. I will not accept late homework except under extraordinary circumstances that you need to discuss with me before the deadline.

Problem 1. A group G is called *metabelian* if it has a normal abelian subgroup $A \trianglelefteq G$ such that G/A is also abelian. Show that any subgroup of a metabelian group is metabelian. (*Hint:* use the Second Isomorphism Theorem.)

Problem 2. Let $G = \mathrm{SL}_2(\mathbf{R})$ denote the group of 2×2-matrices with determinant 1 and let $\mathbf{H} = \{z \in \mathbf{C} \mid \Im(z) > 0\}$ be the upper half-plane of complex numbers with positive imaginary part. Show that G acts on $\mathbf{H}$ by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot z := \frac{az+b}{cz+d}.$$

Compute the kernel of this action and the stabilizer of $z = i$.

Problem 3. Read about conjugacy in symmetric groups, p. 125–126. Determine the sizes of all conjugacy classes of S_5 (given in the table at the top of p. 127) and verify that the class equation holds.

(Do not write a list of all the 120 elements. Instead, give a short argument why the sizes of the classes are what you claim.)